LYAPUNOV STABILITY ANALYSIS OF LARGE SCALE POWER SYSTEMS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By
VIJAY VITTAL

to the
DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
JULY, 1979

EE-1979-M-VIT-LYA

DEDICATED TO

MY LOVING PARENTS

'SHAKKU' AND 'PADDU'

CERTIFICATE

Certified that this work, 'Lyapunov Stability Analysis of Large Scale Power Systems', by VIJAY VITTAL, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

Dr. M.A. PAI
Professor
Department of Electrical Engineering
Indian Institute of Technology
Kanpur

ACKNOWLEDGEMENTS

I acknowledge with gratitude the invaluable guidance, encouragement and inspiration received from my guide and mentor Dr. M.A. PAI who was largely responsible for creating my interest in nonlinear stability theory and its applications, and was an unfailing source of advice, informal discussion and critical appraisal throughout.

I am greatly indebted to Mr. H.C. KARNICK for a number of illuminating discussions held during the course of this project.

I am thankful to Dr. K.R. PADIYAR for some useful discussions in connection with this work. Also my thanks are due to all my fellow research scholars especially Mrs. K. GOMATHI and M/S. R.P. ADGAONKAR, C. RADHAKRISHNA and T.V. PRABHAKAR for discussions in the related topics.

Lastly my thanks to M/s. S.V. SIRBHAI and K.N. TEWARI for their patient and skilful typing.

TABLE OF CONTENTS

The quality that can be defined is not the Absolute Quality
The names that can be given to it are not Absolute names
It is the origin of heaven and earth
When named it is the mother of all things . . .

Tao Te Ching
of
Lao Tzu

CHAPTER I

INTRODUCTION

1.1 POWER SYSTEM STABILITY

The term 'Stability' when used in the context of a power system is that property which ensures that the system will remain in operating equilibrium through the normal and abnormal operating conditions [1]. As the structure of power systems grows larger and more complex, the stability studies gain paramount importance as far as the safety of the total system is concerned. With the ever increasing demand for electrical energy and dependence on an uninterrupted supply, the associated requirement of high reliability dictates that power systems be designed to maintain stability under most disturbances, consistent with economy. Furthermore, the degree of stability of a power system, is an important consideration in the planning and designing of new facilities.

Investigation of the disturbed conditions in a power system following a fault is carried out on a mathematical model of the system. It is customary to classify stability [1,2,3], for the purpose of analysis as 'Steady state stability'

and 'Transient stability'. The occurrences disturbing the stable operation of a power system can be small and gradual changes, as for instance controlled changes of power input and output, or large changes in operating conditions such as uncontrollable loss of large amounts of generation or load, or the occurrence of heavy transmission line faults. The terms gradual and sudden should be interpreted here in relation to the major time constants of the system or part of the system under study. Small and gradual changes constitute problems in steady state stability while large and sudden changes lead to problems in transient state stability.

1.2 TRANSIENT STABILITY

Transient stability as referred to a single synchronous machine connected to an infinite bus may be defined [4] as follows.

DEFINITION: 'If a synchronous machine operating in steady state torque equilibrium issubjected to a disturbance of any kind which results in speed deviations (oscillations) of the machine rotor from the (synchronously rotating) reference axis, then the machine is called transiently stable if the rotor swings die out and the rotor reaches a new stable equilibrium position'.

Transient stability for a multimachine system may be defined [4] as follows.

DEFINITION: 'If individual machines in a multimachine system are operating in a steady state torque equilibrium and a disturbance of any kind is imposed on the system, then the system is called transiently stable if each machine oscillates around and ultimately comes to rest at a new stable torque equilibrium point'.

The large perturbation which creates the transient stability problem, may be sudden increase in load, or a sudden increase in reactance of the circuit, caused for instance by a line outage. But the most severe perturbation to which a power system can be subjected, is a short circuit, generally termed as a 'fault', and a symmetrical three phase short circuit is the most severe of all faults. In designing and planning a system all possible contingencies including the worst conditions have to be considered. Hence in stability studies a three phase short circuit is generally considered [2].

In faulted systems, it is essential to determine when the fault has to be cleared without the rotating machines losing synchronism. The maximum time through which the fault can be left on the system so that the rotating machines will return to their normal conditions after the fault is cleared (by opening one or more circuit breakers) is called the 'critical clearing time'. Determination of critical clearing time constitutes an important aspect of transient

stability study since it is this information which enables the setting of relays, which in turn control the tripping of the circuit breakers.

1.3 CONVENTIONAL METHODS FOR TRANSIENT STABILITY ANALYSIS

The analysis of transient stability is formulated as follows: Given a system does there exist an equilibrium state of the system after the disturbance is cleared? If yes, what is the maximum time that the disturbance may be allowed to remain without the system losing synchronism? The analysis consists of two main steps.

STEP 1: The study of the evolution of the system from the occurrence of the disturbance to the time of clearing.

STEP 2: The study of the evolution of the system beyond the time of clearing.

The intervals of time corresponding to these two steps are called the 'faulted state' and the 'postfault state' respectively. Each of these steps involves the integration of a large number of nonlinear differential equations.

The classical method of stability investigation consists in assuming a clearing time 't_e' arbitrarily and solving the differential equations of the system in its faulted state upto t_e, and in the postfault state after t_e to obtain the swing curves. The initial conditions are

obtained from a prefault load flow data. The swing curves which are a plot of the rotor angles versus time are examined for the stability of the postfault system. This is usually done by observing whether the rotor angle differences with respect to any one machine taken as a reference tend to remain constant during the first swing of the rotor angles. In such a case we conclude that the system is stable. If they tend to diverge, then it is considered to be unstable. Generally it is assumed that if the system is stable during the first swing, it is also stable thereafter. This process is repeated for another clearing time. After a certain number of such repeat procedures, the clearing time at which the transition from the stable to the unstable state of the system takes place is established. This gives the critical clearing time 't_c'. This procedure of determination of the critical clearing time is not efficient computationally when one requires a quick picture of the stability with respect to various possible perturbations. It is because of this reason, that special emphasis has been laid on examining alternative approaches such as direct analytical methods, in the last decade.

Stability investigations by direct methods in contrast to the classical methods are carried out by making partial or no use of the system differential equations. The equal area criterion [2] is a graphical

procedure for single and two machine systems and does not make use of the differential equations as such. Analytical methods such as the phase plane technique [5] were employed for single and two machine systems. The energy integral criterion [6] and the direct method of Lyapunov [7-28] for multimachine systems make use of the system differential equations partially. The Lyapunov's direct method has been a topic of active research in recent years. This method essentially consists in establishing a region of stability around the postfault equilibrium state via the construction of a Lyapunov function. The system is considered stable if the solution trajectories of the faulted and the postfault states lie entirely in this region [8].

1.4 APPLICATION OF LYAPUNOV'S DIRECT METHOD TO THE TRANSIENT STABILITY PROBLEM OF POWER SYSTEMS

The crux of the problem, in the application of Lyapunov's direct method to the transient stability analysis of multimachine power systems, revolves around the replacement of the step 2 detailed in Section 1.3, by a stability criterion in the form of a Lyapunov function around the postfault equilibrium point. Aylett [6] in 1958, was the first to suggest such a criterion based on energy considerations, but the theory was not sound enough for practical implementation. Pioneering work on the application of Lyapunov's direct method to power system transient stability was done by Gless [7] and El-Abiad and Nagappan [8].

procedure for single and two machine systems and does not make use of the differential equations as such. Analytical methods such as the phase plane technique [5] were employed for single and two machine systems. The energy integral criterion [6] and the direct method of Lyapunov [7-28] for multimachine systems make use of the system differential equations partially. The Lyapunov's direct method has been a topic of active research in recent years. This method essentially consists in establishing a region of stability around the postfault equilibrium state via the construction of a Lyapunov function. The system is considered stable if the solution trajectories of the faulted and the postfault states lie entirely in this region [8].

1.4 APPLICATION OF LYAPUNOV'S DIRECT METHOD TO THE TRANSIENT STABILITY PROBLEM OF POWER SYSTEMS

The crux of the problem, in the application of Lyapunov's direct method to the transient stability analysis of multimachine power systems, revolves around the replacement of the step 2 detailed in Section 1.3, by a stability criterion in the form of a Lyapunov function around the postfault equilibrium point. Aylett [6] in 1958, was the first to suggest such a criterion based on energy considerations, but the theory was not sound enough for practical implementation. Pioneering work on the application of Lyapunov's direct method to power system transient stability was done by Gless [7] and El-Abiad and Nagappan [8].

The approach consists of writing the system differential equations in the postfault state (after the final switching) in the form

$$\underline{\dot{x}} = \underline{f}(\underline{x}) \tag{1.1}$$

with the postfault equilibrium state at the origin $\underline{O}$, and seeking a suitable Lyapunov function $V(\underline{x})$ which, along with its time-derivative $\dot{V}(\underline{x})$ has the required sign-definite properties. The stability region around the postfault equilibrium state $\underline{O}$ is specified by an inequality of the type $V(\underline{x}) < C$, where C is a constant to be determined. This constant is usually obtained by evaluating the function $V(\underline{x})$ at the unstable equilibrium point closest to the postfault stable equilibrium point. Now the critical clearing time can be calculated by numerical integration of the faulted system differential equations, substituting at each step the value of $\underline{x}$ into V and checking whether $V < C$. When V reaches the value C (i.e., the faulted system state trajectory reaches the boundary $V = C$), the time at that instant is the estimated critical clearing time 't_c'. Thus an explicit integration of the differential equations is performed only once, resulting in a marked reduction in the computation time, especially in the analysis of large scale power systems for various contingencies. Another advantage of this method lies in investigating the effects of parameter variations such as damping [9,26], power input [27] etc.

It should however be noted that Lyapunov theorems provide only sufficient conditions, and not 'necessary and sufficient' conditions, for stability. Thus, the estimation of stability regions and critical clearing time based on Lyapunov's direct method may turn out to be conservative, but it can be used very effectively as a preliminary screening procedure, and reduce the number of studies to be carried out in detail.

1.5 THE LARGE SCALE POWER SYSTEM STABILITY PROBLEM

The previous section explains the technique of applying Lyapunov methods. Although, concentrated efforts have been made towards the improvement of the Lyapunov functions [12,17], the application of the theory to realistic systems has been very slow. This is due to the fact that there are a few drawbacks, apart from the inherent conservativeness of the method. They are (i) The application of the theory to large power systems poses heavy computational problems in the determination of stability regions based on computing $V(\underline{x})$ at the nearest unstable equilibrium point (u.e.p.) [28]. It has been established that for a n-machine system, the number of unstable equilibrium points is $2^{n-1}-1$, and consequently very large when 'n' is large. This problem has received considerable attention recently [21, 22], but requires further refinement to obtain more accurate stability regions. (ii) The selection of the

models for the system, is strongly constrained by the possibility of a finding a suitable Lyapunov function [28]. It has not been possible to include refinements like accurate models of synchronous machines, saturation effects etc., into the Lyapunov functions for multimachine systems. Most of the work is confined to classical models with other effects included partially. (iii) As size of the system grows the overall computational burden in using Lyapunov functions also increases.

The above considerations therefore call for new theoretic approaches to the problem. Two such promising approaches are:

(i) Dynamic equivalencing [29-34]

(ii) Use of vector Lyapunov functions [36-59].

The first approach consists in obtaining a simplified model after coherency analysis and dynamic equivalencing. Lyapunov functions are then constructed for this reduced order model.

The second method uses a theoretical approach which was first proposed by Bellman [37] and Bailey [36]. They demonstrated its usefulness for the stability analysis of a complex composite system. Most complex systems are made up of a number of low order interacting systems. These low order systems in hierarchial theory [38] are called 'subsystems'. These subsystems are arrived at on the basis of decomposition of the large system.

Today's power systems being large and complex become natural candidates for the application of the decomposition - aggregation technique. The technique consists in decomposing the composite system into low order subsystems. Suitable Lyapunov functions are constructed in the usual manner for these systems, satisfying the required sign definite properties. On a higher hierarchial level, the concept of a vector Lyapunov function is introduced in which these scalar functions form the elements of the vector Lyapunov function defined for the composite system. Using the properties of interactions among the subsystems, conditions and region of stability for the overall system are derived. Initially this technique was restricted to a particular class of interconnections viz. linear interconnections. Nonlinear interconnections were proposed by Piontkovskii et al. [39]. Recent works of Šiljak [40-43], Grujić and Šiljak [44-45], Thompson [76], Araki and Kondo [47], Michel [48-50], Pai and Narayana [51], Jocić, Ribbens Pavella and Šiljak [52], and Jocić and Šiljak [53] give an explicit exposition of these concepts, and provide an impetus to apply the results to large scale power systems.

1.6 TRANSFER CONDUCTANCES IN MULTIMACHINE POWER SYSTEMS

In the application of Lyapunov's method to realistic systems there arises a difficulty of taking into account the effect of the transfer conductances contained in the system.

In early investigations, transfer conductances were entirely neglected or only partially taken into account [8,12]. Although the conductance components take small values as compared with the susceptance components, it still remains an open problem whether or not we can neglect the conductance components from the standpoint of the transient stability. For certain cases, as shown by Ribbens-Pavella [14], the neglect of the transfer conductances may be justified. However, that may not be the case in general. Especially, they cannot be neglected if the system is heavily loaded. It has been pointed out by Uemara et al. [60] that there exists a danger in judging a practically unstable system to be stable if we use a Lyapunov function which neglects the transfer conductances.

In recent works [61-62] methods have been proposed of including transfer conductances for transient stability analysis.

1.7 OBJECTIVES AND OUTLINES OF THE THESIS

The objectives of the thesis are:

i) Application of Lyapunov methods to large scale power systems using dynamic equivalencing.

ii) Investigation of new decomposition structures for vector Lyapunov function application to power systems and validating on a multimachine power system.

iii) Examining effects of transfer conductances on the critical clearing time.

In Chapter II, Lyapunov's method for a realistic power system consisting of 13 machines, 71 buses and 94 lines is applied for the full system as well as for the reduced system obtained by dynamic equivalencing procedure. For the latter, the system is divided into the study system and the external system. Coherency analysis is then done for generators in the external system . The coherent groups are equivalised using the method proposed by Podmore [29]. Lyapunov method is then applied to the reduced system. Critical clearing times are computed for various fault locations in the study system and the results obtained are compared with those using Lyapunov method for the full system as well as with those obtained by a base case transient stability analysis.

Chapter III presents a new decomposition technique based on the 'centre of angle' principle proposed by Stanton [54] and Tavora and Smith [55], for the application of vector Lyapunov methods to large scale power systems. A vector Lyapunov function for the composite system is then found, and an overall stability region is constructed using the separate results of Walker and McClamroch [59], Weissenberger [56], Grujic and Siljak [45], Araki [57] and Michel [48]. A four machine power system considered earlier by El-Abiad and Nagappan [8] is worked out using this method and the results are quite encouraging.

In Chapter IV Lyapunov functions containing partially the effects of the transfer conductances, as proposed by

Pai and Varwandkar [62] are used in stability analysis of the UPSEB system, and the critical fault clearing times are calculated. Comparison is made with the critical clearing times obtained without the transfer conductances.

In the concluding chapter, the results of the thesis are reviewed and problem for future research outlined.

Your knowledge is your friend in distant lands.
Your wife is your friend at home.
To the sick, the right medicine is the friend.
Dharma is a friend even beyond the graves.

'VEDAS'

CHAPTER II

DYNAMIC EQUIVALENCING AND APPLICATION OF LYAPUNOV'S METHOD TO A PRACTICAL POWER SYSTEM

2.1 INTRODUCTION

During the last decade, several research workers have applied Lyapunov's direct method for the analysis of the stability of power systems following a sudden disturbance. It still continues to be a fertile area for research for power engineers all over the world because of the yet unsolved problems seeking solutions. One of the problems is the dimensionality of the large scale systems. In spite of improved Lyapunov functions being available, computing effort is still considerable due to the need for computing efficiently the stability regions. These regions are computed by evaluating $V(\underline{x})$ at the nearest u.e.p. One of the ways to reduce computational time in evaluating u.e.p. is through dynamic equivalencing procedures and model reduction techniques.

In this chapter Lyapunov's direct method is applied to the UPSEB system consisting of 13 machines, 71 buses and

94 lines using dynamic equivalencing procedure. Coherency analysis is first done for the external system, and the group of coherent generators is found. These generators are then equivalenced using a method proposed by Podmore [29-31]. Lyapunov's method is then applied to the reduced order model, and critical clearing times are evaluated for a set of representative fault locations in the study system. Critical clearing times for the same set of faults are computed for the original system using Lyapunov's method. A comparison of the two results, validates the dynamic equivalent, and also helps to prove the feasibility of applying Lyapunov's method to reduced order models. There is a considerable saving in computer time through this procedure.

2.2 MATHEMATICAL MODEL OF THE POWER SYSTEM

A number of simplifying assumptions are made in arriving at a mathematical model for which satisfactory Lyapunov functions can be constructed. These are:

(i) The transmission network is considered to be in steady state which results in an amenable analytical expression for the electrical power P_{ei} delivered by each generator.

(ii) The voltage behind the transient reactance is assumed constant. In other words it is assumed that the flux linkages of various machines are constant.

(iii) Damping is assumed to be directly proportional to the slip velocity and is thus mainly due to the mechanical friction and asynchronous torques.

(iv) The mechanical power input to each machine during disturbance is assumed constant and equal to the input prior to occurrence of the fault. This assumption is justified because the governor time constants are much larger than the duration of the transient swings.

(v) Loads are represented as constant impedances which results in the overall nodal admittance matrix at the internal nodes of the generator being constant.

(vi) Transfer conductances in the admittance matrix are usually neglected.

(vii) Saturation is neglected.

(viii) Only round rotor machines are considered thus neglecting the effect of saliency.

Under the above assumptions the equation governing the rotor motion of each machine is given by

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_{mi} - P_{ei} , \quad i=1,2,\ldots,n \tag{2.1}$$

where

M_i - inertia constant of the ith machine

D_i - damping coefficient of the ith machine

δ_i - rotor angle measured with respect to a synchronously rotating reference frame

P_{mi} - mechanical power input to the ith machine

P_{ei} - electrical power delivered by the ith machine.

The electrical power P_{ei} delivered by the ith machine is given by

$$P_{ei} = E_i^2 G_{ii} - \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j Y_{ij} \operatorname{Cos}((\delta_i - \delta_j) - \theta_{ij}) \qquad (2.2)$$

$$i=1,2,\ldots,n$$

where

E_i = - internal voltage

$Y_{ij} = G_{ij}+jB_{ij}$ - the transfer admittance between the ith and the jth machines

$\theta_{ij} = \tan^{-1}(B_{ij}/G_{ij})$ - the admittance angle between the ith and the jth machines.

Substitution of (2.2) into (2.1) yields

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = P_{mi} - E_i^2 G_{ii} - \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j Y_{ij} \operatorname{Cos}(\delta_{ij}-\theta_{ij})$$

$$i=1,2,\ldots,n \qquad (2.3)$$

Use of the appropriate values of Y_{ij}, θ_{ij} in (2.3) render them valid either for the faulted or the postfault state. The postfault steady state δ_i^o is given by the solution of (2.3) with Y_{ij}, θ_{ij} pertaining to the postfault network conditions and $\ddot{\delta}_i = 0$, $\dot{\delta}_i = 0$. This implies that the angles δ_i^o $(i = 1,2,\ldots,n)$ constitute a solution of the equations

$$P_{mi}-E_i^2 G_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j Y_{ij} \operatorname{Cos}(\delta_{ij} - \theta_{ij}) \qquad i=1,2,\ldots,n \qquad (2.4)$$

Since δ_i^o satisfies Eq.(2.4) the swing equations (2.3) can be expressed as

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = - \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j Y_{ij} \quad Cos(\delta_{ij}-\theta_{ij})-Cos(\delta_{ij}^o-\theta_{ij})$$

$$i=1,2,\ldots,n \qquad (2.5)$$

Expanding the right hand side trigonometrically and using the relation $G_{ij} = Y_{ij}$ Cos θ_{ij} and $B_{ij} = Y_{ij}$ Sin θ_{ij}, we get

$$M_i \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} = - \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j \quad G_{ij}(Cos\ \delta_{ij}-Cos\ \delta_{ij}^o)$$

$$+ B_{ij}(Sin\ \delta_{ij}-Sin\ \delta_{ij}^o) \quad , i=1,2,\ldots,n$$

$$(2.6)$$

The equation (2.6) can be expressed in the following state variable form

$$\dot{\underline{x}} = A\underline{x} - B\ f(\underline{\sigma})$$
$$\underline{\sigma} = C\ \underline{x} \qquad (2.7)$$

The formation and structure of matrices A, B and C for any given system is discussed in references [21,32].

2.3 STEPS INVOLVED IN APPLYING LYAPUNOV'S METHOD TO A POWER SYSTEM

The procedure for applying Lyapunov's method to a power system and determining the critical clearing time, consists of six main steps.

1. Load flow for the prefault system.
2. Determination of driving point and transfer admittances between the internal buses of the machines for the faulted and postfault system.
3. Determination of the postfault stable equilibrium point and testing its stability.
4. Construction of a Lyapunov function (V-function) about the postfault stable equilibrium point of the system.
5. Determination of the boundary $V < C$ of the stability region, where C is the value of the V-function at the unstable equilibrium point closest to the stable equilibrium point for the postfault system.
6. Integration of the faulted system equations, until trajectory reaches the boundary. This gives the critical clearing time.

2.4 THE STABLE EQUILIBRIUM POINT

This is a very important step in the formulation of the stability criterion. The postfault system differential equations are given by Eq.(2.1). Equations (2.1) can be cast in the following state variable form.

$$\dot{\underline{x}} = \underline{F}_1(\underline{x}) \tag{2.8}$$

The equilibrium states of the postfault system can be found by solving the following nonlinear algebraic equation

$$\underline{F}_1(\underline{x}) = 0 \tag{2.9}$$

The stable solution of (2.9) is given by minimizing $\underline{F}_1^T \underline{F}_1$ in the vicinity of the prefault stable solution by any well known method like the Fletcher-Powell or conjugate gradient method. The minimization problem becomes time consuming and complicated (in the sense of convergence) as the size of the system (i.e. number of state variables) increases. To eliminate the unwieldy minimization procedure, a new special load flow technique proposed in ref. [24] has been used to compute the postfault stable equilibrium point.

Having reduced the system to generator internal nodes, each node except one which is the slack node is treated as a 'voltage controlled node' (P-V node). The node which corresponds to the machine having maximum generation at the prefault stage is generally chosen as the slack node. The solution of the load-flow problem is initiated by assuming voltage angles at each node equal to the prefault steady state angles, and voltage magnitudes equal to the scheduled voltages. Therefore voltage at the pth node will be

$$V_p^o = |E_{sh,p}| (\text{Cos } \delta_p + j \text{ Sin } \delta_p) \; ; \quad p=1,2,\ldots,n \tag{2.10}$$

where $|E_{sh,p}|$ is the scheduled voltage at the pth node. The complex power flowing into the pth node is given by

$$P_p - jQ_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q \tag{2.11}$$

It is to be noted in equation (2.11) that P_p is equal to P_{mp}

i.e. power delivered by the machine connected to the pth node. The reactive power into the pth node can be calculated from equation (2.11)

$$Q_p = -\mathrm{Im}\left[V_p^* \sum_{q=1}^{n} Y_{pq} V_q\right] \tag{2.12}$$

The voltage V_p must satisfy the relation

$$|V_p| = |E_{sh,p}| \tag{2.13}$$

in order to calculate the reactive node power required to provide the scheduled node voltage. The present estimate V_p^* must be adjusted, therefore to satisfy Eq.(2.13). The phase angle of the estimated node voltage is

$$\delta_p^k = \tan^{-1} \frac{\mathrm{Im}(V_p^k)}{\mathrm{Re}(V_p^k)} \tag{2.14}$$

Assuming that the angles of the estimated and scheduled voltages are equal, the adjusted estimate for V_p^k is

$$V_{p(new)}^k = |E_{sh,p}| \,(\mathrm{Cos}\,\delta_p^k + j\mathrm{Sin}\,\delta_p^k) \tag{2.15}$$

Substituting $V_{p(new)}^k$ in eq.(2.12) the reactive power Q_p^k is obtained. The new voltage estimate V_p^{k+1} is calculated using Gauss-Seidel iterative scheme, by the following relation

$$V_p^{k+1} = \frac{1}{Y_{pp}}\left[\frac{(P_p - jQ_p^k)}{(V_p^k)^*_{new}} - \sum_{q=1}^{p-1} Y_{pq} V_q^{k+1} - \sum_{q=p+1}^{n} Y_{pq} V_q^k\right] \tag{2.16}$$

Similarly new estimates for all the nodes V_i^{k+1} ($i=1,2,\ldots,n$; $i\neq S$, where S is the slack node) are obtained.

The rate of convergence can be increased by applying an acceleration factor to the approximate solution obtained for each iteration. Let α and β be the acceleration factors respectively for the real and imaginary components of voltage. The accelerated value is

$$V^{k+1}_{p(\text{accelerated})} = V_p^k + \alpha \,\text{Re}\,[\,\bar{V}_p^{k+1} - V_p^k\,] + j\beta \,\text{Im}\,[\,\bar{V}_p^{k+1} - V_p^k\,] \quad (2.17)$$

and replace the calculated V_p^{k+1} by $V^{k+1}_{p(\text{accelerated})}$. The entire process is iterated till

$$\max_p \left|\, |V_p^{k+1}| - |E_{sh,p}| \,\right| \leqslant \varepsilon \quad p=1,2,\ldots,n;\ p\neq S \quad (2.18)$$

is satisfied. The angles of these converged voltages are the desired stable equilibrium angles for postfault state.

It is to be noted that since all the nodes of the reduced system are voltage controlled nodes, the deviations in the node reactive powers are insignificant during iterative process. Hence there is no need to keep a continuous watch on the node reactive powers to check whether they cross the reactive power limit.

2.5 UNSTABLE EQUILIBRIUM POINTS AND REGION OF STABILITY

An unstable equilibrium point of Eq.(2.8) may be determined by minimizing $\underline{F}_1^T\, \underline{F}_1$ by choosing a proper starting

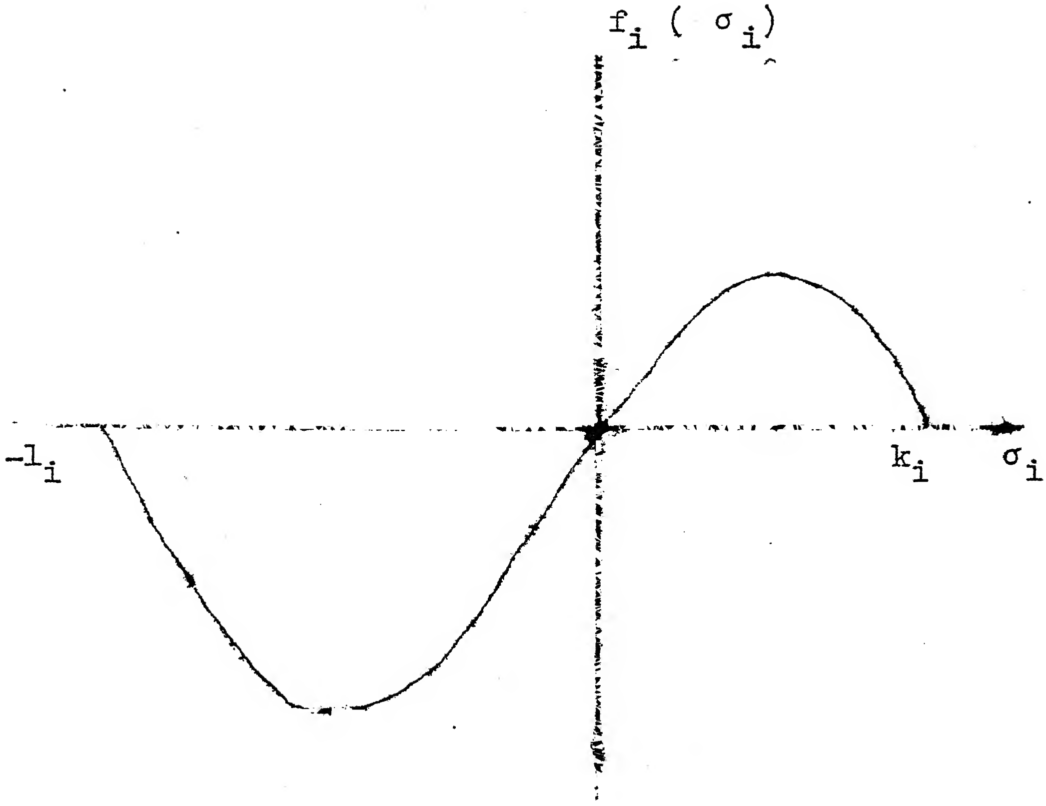

FIG .2.1: THE POWER SYSTEM NONLINEARITY

point. It can be readily seen that the determination of $(2^{n-1}-1)$ unstable equilibrium points becomes increasingly unmanageable as system size (n) increases. In fact this is one of the biggest obstacles in the use of Lyapunov stability criterion for power systems. However, an accurate determination of the unstable equilibrium point is not necessary from the engineering point of view as the value of $V(\underline{x})$ is not very sensitive to small changes in $\underline{x}$ around the equilibrium points [23]. A method proposed by C.L. Narayana [22] known as the corner point method takes advantage of this fact in the calculation of the region of asymptotic stability.

Consider a n-machine power system. Fig.2.1 shows the power system nonlinearity. For the system considered, there are ${}^nC_2 = n(n-1)/2$, say m, nonlinearities in the system. Each nonlinearity represents power transfer between two machines, and is a function of one of the rotor angular differences. Out of m angular differences only (n-1) can be selected independently. Therefore $\underline{\sigma}$ defined by (2.7) is partitioned as

$$\underline{\sigma} = \begin{bmatrix} \underline{\sigma}_I \\ \underline{\sigma}_D \end{bmatrix} \tag{2.19}$$

Each $f_i(\sigma_i)$, $i \in I$, violate the sector conditions for $l_i > \sigma_i > k_i$. Let $\underline{\sigma}_I$ be considered in a (n-1)-dimensional space $R^{(n-1)}$, such that its axis represents σ_i. Two

hyperplanes $\sigma_i = k_i$ and $\sigma_i = l_i$ restrict the space $R^{(n-1)}$ around the origin in which $f_i(\sigma_i)$, $i \in I$, satisfies the sector conditions. Similarly $2(n-1)$ hyperplanes can be drawn to form a polyhedron. A polyhedron, so formed, has 2^{n-1} corner points and it is claimed that the minimum value of $V(\underline{x})$, obtained by a search among 2^{n-1} corner points, yields an approximate region of stability [23]. The values of k_i and l_i are determined by the following relations

$$k_i = \pi - 2(\delta_p^o - \delta_q^o)$$

$$l_i = -\pi - 2(\delta_p^o - \delta_q^o) \quad : \quad p < q \tag{2.20}$$

where $(\delta_p^o - \delta_q^o)$ is the angular difference of pth and qth machines at the stable equilibrium point and it is associated with σ_i, $i \in I$.

For a chosen $\underline{\sigma}_I$, a polyhedron can be constructed. Since there are ${}^mC_{n-1}$, say p, ways to choose $\underline{\sigma}_I$, p polyhedrons can be formed and a search among $p.2^{n-1}$ corner points has to be carried out. Such an enumeration and computation for practical systems is difficult to implement.

To eliminate the various choices of $\underline{\sigma}_I$, a machine, say r, is selected as reference machine, the choice of linearly independent variables of $\underline{\sigma}_I$ is made in such a way that σ_i, $i = 1,2,\ldots,(n-1)$, involves an angular difference which contains the term δ_r. This results in a set $\underline{\sigma}_I$, which defines

the boundary of the polyhedron, and has 2^{n-1} corner points. The choice of the corner point which results in a minimum value of Lyapunov function among the 2^{n-1} such points is made on the basis that each component σ_i corresponds to $\min \{|k_i|, |l_i|\}$. Thus, for a machine taken as reference, a unique corner point is determined. The entire process is iterated for all machines taken as reference machine. Hence the search is reduced to n corner points. The minimum of the V-function evaluated at n such points, gives the region of stability.

The main steps to calculate the region of stability can be summarised as follows:

1. Calculate all the possible angular differences $(\delta_i^o - \delta_j^o)$, $i = j = 1, 2, \ldots, n$, $i \neq j$.
2. Choose a reference machine, say r.
3. Among all the angular differences of step 1, pick all the (n-1) differences $(\delta_i^o - \delta_j^o)$, such that i or j = r, hence form $\underline{\sigma}_I$.
4. Construct the polyhedron using (2.20).
5. Choose a corner point which corresponds to $\min_i \{|k_i|, |l_i|\}$, $i = 1, 2, \ldots, (n-1)$.
6. Evaluate $V(\underline{x})$ if corner point selected in step 5 does not violate sector conditions.
7. Repeat steps 2 to 6 for all machines taken as reference machine.

8. Minimum of all values of $V(\underline{x})$, say C, obtained in step 6, gives the region of stability : $V(\underline{x}) < C$.

2.6 LYAPUNOV FUNCTION

In this analysis, a Lyapunov function based on the total system energy concept has been used. This function was proposed by El-Abiad and Nagappan [8], but was later modified by Willems and Willems [15], because the transfer conductance term in it caused the function to have an indefinite time derivative around the origin. The function is given by the expression

$$V(\underline{x}) = \sum_{i=1}^{n} \frac{M_i \omega_i^2}{2} + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} E_i E_j B_{ij} [\,\mathrm{Cos}(\delta_i^o - \delta_j^o) - \mathrm{Cos}(\delta_i - \delta_j) - (\delta_i - \delta_j - \delta_i^o + \delta_j^o)\, \mathrm{Sin}(\delta_i^o - \delta_j^o)\,] \tag{2.21}$$

This completes the procedure detailing the application of Lyapunov's direct method to power systems.

2.7 DYNAMIC EQUIVALENCING PROCEDURE

The method proposed by Stanton, Podmore and Germond[29-31] is adopted. The development of their equivalent is systematic and general, accommodating different types of machines and their control equipments. The basic steps in the development are:

(i) Identification of coherent generators.

(ii) Reduction of coherent generator terminal buses.

(iii) Dynamic aggregation of coherent generators.

(iv) Reduction of load buses.

2.8 IDENTIFICATION OF COHERENT GENERATORS

This is a key step in the techniques using coherency based dynamic equivalents.

Definition of Coherent Generators:

Two generators 'i' and 'j' are said to be coherent if their angular difference during the transient period is constant to a specified tolerance

$$|\delta_i(t) - \delta_j(t)| < \varepsilon \tag{2.22}$$

where ε is the tolerance. A group of generators is said to be coherent if every pair of generators in the group is coherent.

In this thesis a method proposed by Pai and Adgaonkar[33] is used to identify the coherent generators. This method is an improvement over Podmore's linear simulation [34]. The technique of identifying the coherent generators is detailed as follows:

The power system is divided into two parts namely (i) study system, in which the system behaviour is of direct interest and where a set of fault locations are considered and (ii) external system, which is equivalised. The linearised swing equations are

$$\frac{d\,\Delta\omega_i}{dt} + D_i\frac{\Delta\omega_i}{dt} = P_{mi} - P_{ei} \tag{2.23}$$

$$\frac{d\,\delta_i}{dt} = \Delta\omega_i \qquad i = 1,2,\ldots,n$$

In order to eliminate $\Delta\underline{P}_e$ from equation (2.23), the network equations are modified as

$$\begin{bmatrix} \Delta\underline{P}_e \\ \\ \Delta\underline{P}_l \end{bmatrix} = \begin{bmatrix} \underline{\hat{H}}_{GG} & \underline{\hat{H}}_{GL} \\ \\ \underline{\hat{H}}_{LG} & \underline{\hat{H}}_{LL} \end{bmatrix} \begin{bmatrix} \Delta\underline{\delta}_n \\ \\ \Delta\underline{\phi}_n \end{bmatrix} \tag{2.24}$$

where $\hat{H}_{GG}$, $\hat{H}_{GL}$, $\hat{H}_{LG}$ and $\hat{H}_{LL}$ are the partial derivatives of active power with respect to the bus voltage angles.

δ = angles at the generator internal bus

ϕ = angles at the load buses.

where

$$\Delta\underline{\delta}_n = \left[\Delta\delta_{1n}, \Delta\delta_{2n}, \ldots, \Delta\delta_{n-1,n}\right]$$

$$\Delta\underline{\phi}_n = \left[\Delta\phi_{n+1,n}\ \Delta\phi_{n+2,n}, \ldots, \Delta\phi_{n+k,n}\right]$$

'n' is the number of generators and 'k' the number of buses. In equation (2.24), all the angles are measured with respect to a generator 'n' which we consider as the reference generator. From equation (2.24), we have

$$\Delta\underline{P}_e = (\hat{H}_{GG} - \hat{H}_{GL}\,\hat{H}_{LL}^{-1}\,\hat{H}_{LG})\Delta\underline{\delta}_n - \hat{H}_{GL}\,\hat{H}_{LL}^{-1}\,\Delta\underline{P}_l \tag{2.25}$$

by eliminating $\Delta \underline{P}_e$ from (2.23) and (2.25) the system equations (2.23) are cast in the usual state space form as

$$\underline{x} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} \quad \text{with} \quad \underline{x}(0) = \underline{0} \tag{2.26}$$

The introduction of a network fault results in the instantaneous application of an accelerating torque to the synchronous generators [31,34]. The electrical power output of the units may be computed by solving the faulted network equations with the voltage behind the generator transient reactances fixed at the prefault values. Therefore, the system models for the faulted and postfault period become

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}\,; \qquad \underline{x}(0) = \underline{0}\,; \quad 0 \leqslant t \leqslant t_e \tag{2.27}$$

$$\dot{\underline{x}} = \underline{A}\,\underline{x} \qquad ; \qquad \underline{x}(t_e) = \underline{x}_e \quad ; \quad t \geqslant t_e \tag{2.28}$$

where $\underline{x}_e$ is the state of the system at $t = t_e$ and is computed from (2.27). Assuming that the system matrix has all the distinct eigenvalues, the response of (2.28) is written as a linear combination of modes as

$$\underline{x} = \sum_{j=1}^{n-2} a_j\, x_j\, e^{\lambda_j t} \tag{2.29}$$

where $\lambda_j = \alpha_j + j\,\beta_j$ = eigenvalue of $\underline{A}$

$\underline{X}_j = \underline{X}'_j + j\underline{X}''_j$ = eigenvector corresponding to λ_j

$a_j = a'_j + ja''_j$ = scalar which depends upon the reciprocal basis vectors to $\underline{X}_j$ and initial conditions $\underline{x}_e$.

Let there be 'm' pairs of complex and (2n-2-2m) real eigenvalues. The response (2.29) is written as

$$\underline{x} = \sum_{j=1}^{m} e^{\alpha_j t}(\underline{b}_j' \, \mathrm{Cos}\ \beta_j t + \underline{b}_j'' \, \mathrm{Sin}\ \beta_j t) + \sum_{j=2m+1}^{2n-2} \underline{b}_j e^{\alpha_j t} \tag{2.30}$$

where

$\underline{b}_j' = a_j' \underline{X}_j' - a_j'' \underline{X}_j''$ $\qquad a_j' = \langle \underline{Y}_i', \underline{x}_e \rangle$

$\underline{b}_j'' = -a_j'' \underline{X}_j' - a_j' \underline{X}_j''$ $\qquad a_j'' = \langle \underline{Y}_j'', \underline{x}_e \rangle$

$\underline{b}_j = a_j' \underline{X}_j'$ $\qquad Y_i' + jY_j'' =$ reciprocal basis vector to $(\underline{X}_i' + j\underline{X}_i'')$

In order to ascertain coherency between the generators 'p' and 'q' we need to examine the maximum angular excursion between the state variables $\Delta\delta_{pn}$ and $\Delta\delta_{qn}$. From (2.30) the expression for $\Delta\delta_{pq} = (\Delta\delta_{pn} - \Delta\delta_{qn})$ is written as

$$\Delta\delta_{pq} = \sum_{j=1}^{m} e^{\alpha_j t} \left[(b_{pj}' - b_{qj}') \mathrm{Cos}\,\beta_j t + (b_{pj}'' - b_{qj}'') \mathrm{Sin}\ \beta_j t \right] + \sum_{j=2m+1}^{2n-2} (b_{pj} - b_{qj})\, e^{\alpha_j t} \tag{2.31}$$

where b_{pj}' and b_{pj}'' are the elements of $\underline{b}_j'$ and $\underline{b}_j''$ respectively, corresponding to the state variable $\Delta\delta_{pn}$.

Now the coherency index h_{pq} which is a normalised bound on the deviation of $\Delta\delta_{pq}$ in the transient period is defined as

$$h_{pq} = \frac{\sum_{j=1}^{m} [(b'_{pj}-b'_{qj})^2+(b''_{pj}-b''_{qj})^2] + \sum_{j=2m+1}^{2n-2} (b_{pj}-b_{qj})^2}{\underset{r}{\text{Max}} [\sum_{j=1}^{m} (b'_{rj}+b''_{rj}) + \sum_{j=2m+1}^{2n-2} b^2_{rj}]} \quad r=1,2,\ldots,n-1 \qquad (2.32)$$

The generators 'p' and 'q' are classified as coherent if

$$h_{pq} < \varepsilon$$

where ε is the tolerance. is generally between $\pm 2.5^\circ$.

2.9 REDUCTION OF COHERENT GENERATOR BUSES

The terminal buses of the generators in a coherent group are replaced by an equivalent bus 't'. The choice of 'V_t' the equivalent bus voltage is flexible, generally it is the average of the individual bus voltages in the group i.e.

$$V_t = \frac{1}{n} \sum_{k=m+1}^{n} |V_k| \qquad (2.34)$$

$$\theta_t = \frac{1}{n} \sum_{k=m+1}^{n} \theta_k$$

where θ_k is the angle of voltage V_k.

For the purpose of clarity, the process of combining the terminal buses of the generators in a group can be interpreted as a series of operations on the physical network as explained below.

(i) Once the voltage V_t is defined as in (2.34), each generator terminal bus in the group is connected to

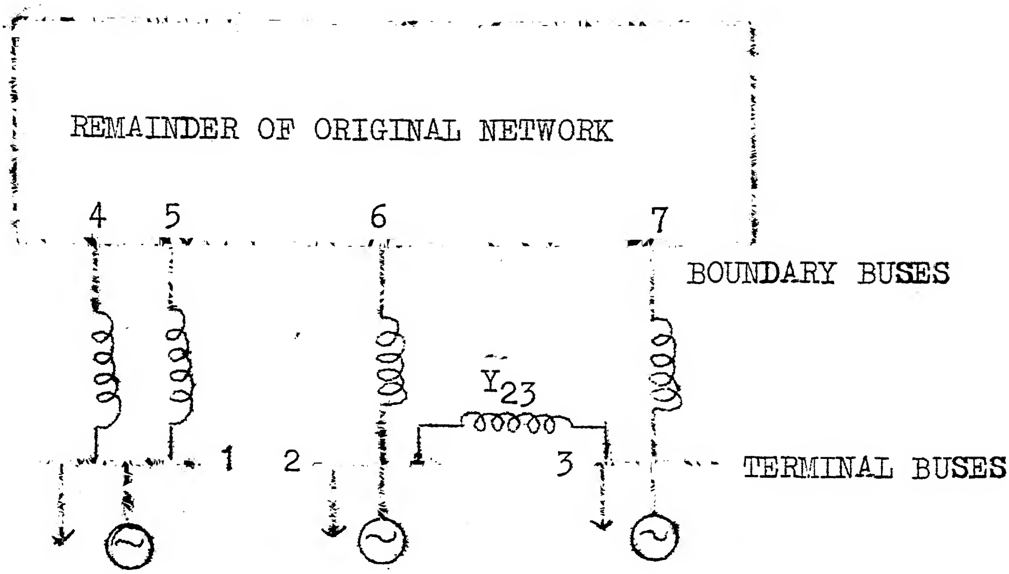

FIG.2.2.a CONFIGURATION OF THE COHERENT GENERATORS IN THE ORIGINAL SYSTEM

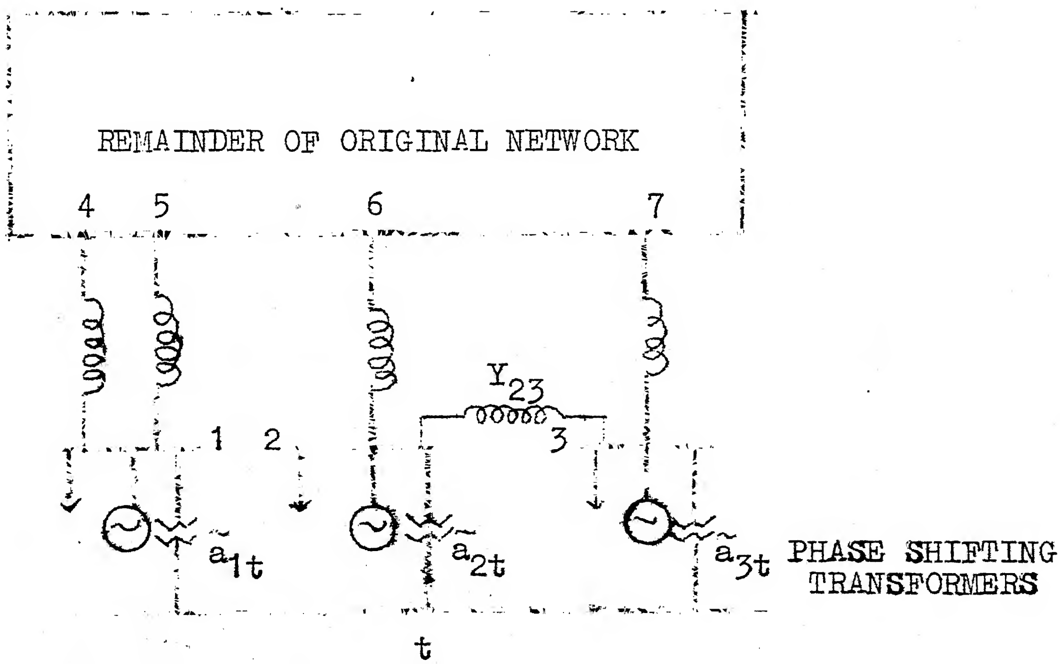

FIG.2.2.b THE COHERENT BUSES ARE CONNECTED TO THE EQUIVALENT BUS THROUGH IDEAL PHASE SHIFTING TRANSFORMERS.

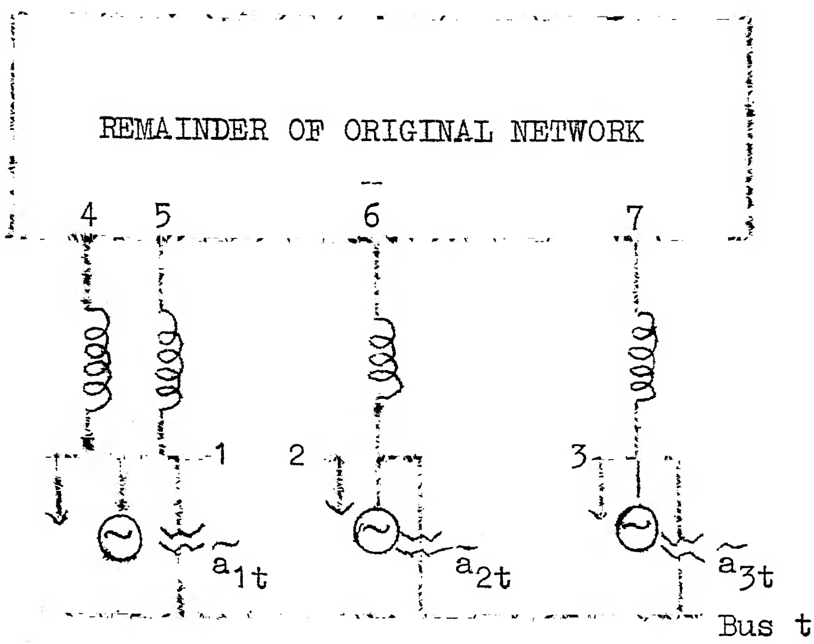

FIG.2.3.c THE BRANCH BETWEEN THE COHERENT BUSES (2) AND (3) IS REPLACED BY THE EQUIVALENT SHUNT ADMITTANCES AT BUSES (2) AND (3).

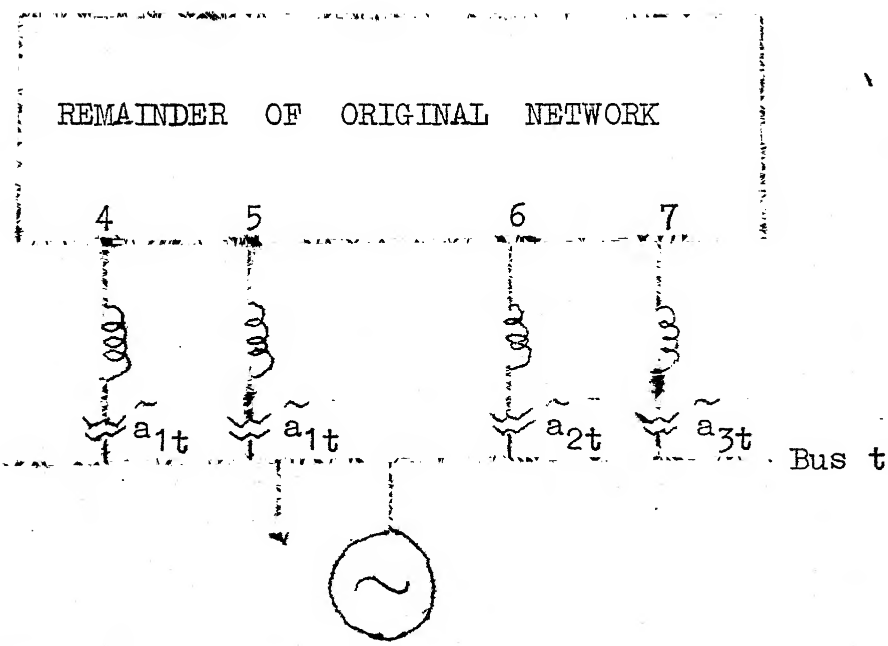

FIG.2.3.d GENERATION, LOADS AND SHUNT ADMITTANCES ARE TRANSFERRED TO BUS t AND THE ORIGINAL TERMINAL BUSES ARE ELIMINATED.

bus 't' through an ideal phase shifting transformer with complex turns ratio $a_b = (V_b/V_t)$, where b is a boundary bus. Under coherent conditions, the ratio a_b is constant and no circulating current flows through the phase shifting transformers. Thus inclusion of the phase shifting transformer does not affect the system (Fig. 2.2b).

(ii) In the second step, the non-radial lines between the coherent generators are detected and are to be replaced. Since there is no change in power flow over these lines during transient period, the lines are replaced by two shunt admittances at the respective end buses (Fig.2.2c) given by

$$y_{bt} = \sum_{k=m+1}^{n} \frac{V_k}{V_t} Y_{bk} \tag{2.35}$$

(iii) The generation, loads, shunt admittances on the terminal buses are transformed to the equivalent bus 't'. The shunt admittances are adjusted to account for the complex turns ratio of the phase shifting transformers. Then the entity of the terminal buses is removed by connecting the phase shifting transformers in series with the respective interconnecting lines (Fig. 2.2d).

2.10 DYNAMIC AGGREGATION OF GENERATING UNITS

The objective of dynamic aggregation is to determine the equivalent generating unit model from the individual generating unit models in the group. The parameters of each of the components of the equivalent generating unit are

obtained by considering the corresponding components of the individual units in the coherent group.

The technique adopted for determining the parameters is to numerically adjust the parameters of the component in the equivalent generating unit, so as to obtain a minimal error between the frequency response of the equivalent transfer function of the component and sum of frequency responses of the invividual transfer functions.

Since, the classical model of the synchronous machine has been considered the procedure of dynamic aggregation will be described only for the rotor dynamics, and synchronous machine.

1. Rotor Dynamics

The swing equation for one machine is given by

$$M_i \frac{d\omega_i}{dt} = P_{mi} - P_{ei} - D_i \omega_i \tag{2.36}$$

Since the machines in a coherent group have nearly equal speed deviations, the summation of the equations (2.36) over the generators in the coherent group yields

$$\sum_j M_j \frac{d\omega_j}{dt} = \sum_j P_{mi} - \sum_j P_{ei} - \left(\sum_j D_j\right)\omega_j \tag{2.37}$$

From the equation (2.37), it follows that (i) the equivalent inertia constant is the sum of the individual inertia constants in the group and (ii) the equivalent damping coefficient is the sum of the individual damping coefficients.

2. Synchronous Machine

The synchronous machine in a coherent group is represented by the classical model of a constant voltage behind transient reactance, hence the terminal voltage is same for each machine in the group, since they are connected in parallel to the same bus after the reduction of the coherent bus. The equivalent reactance is given by the value of the individual reactances taken in parallel.

2.11 REDUCTION OF LOAD BUSES

The loads are treated here as constant impedances (linear characteristics) hence the load buses can be eliminated by Gaussian elimination formula, which is also known as Ward-Hale or Kron's reduction formula.

Elimination Formula

The network equations are written as

$$\begin{bmatrix} I_1' \\ I_2' \\ I_3' \\ \cdot \\ \cdot \\ \cdot \\ I_N' \end{bmatrix} = \begin{bmatrix} Y_{11}' & Y_{12}' & Y_{13}' & \cdots & Y_{1N}' \\ Y_{21}' & Y_{22}' & Y_{23}' & \cdots & Y_{2N}' \\ Y_{31}' & Y_{32}' & Y_{33}' & \cdots & Y_{3N}' \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ Y_{N1}' & Y_{N2}' & Y_{N3}' & \cdots & Y_{NN}' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_2' \\ \cdot \\ \vdots \\ \cdot \\ V_N' \end{bmatrix} \tag{2.38}$$

where N is the total number of buses in the system.

Operating the matrix with standard Gaussian elimination technique for eliminating the variables by columns, at i-1th stage we reach the following structure

$$\begin{bmatrix} I_1^i \\ \vdots \\ I_i^i \\ \vdots \\ I_N^i \end{bmatrix} = \begin{bmatrix} Y_{11}^i & Y_{12}^i & \cdots\cdots & & Y_{1N}^i \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & Y_{ii}^i & \cdots & Y_{iN}^i \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & Y_{Ni}^i & \cdots & Y_{NN}^i \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ \vdots \\ V_N \end{bmatrix} \qquad (2.39)$$

Extracting i, i+1 , ..., N rows from (2.39), we obtain

$$\begin{bmatrix} I_i^i \\ \vdots \\ I_N^i \end{bmatrix} = \begin{bmatrix} Y_{ii}^i & \cdots & Y_{iN}^i \\ \vdots & & \vdots \\ Y_{Ni}^i & \cdots & Y_{NN}^i \end{bmatrix} \begin{bmatrix} V_i \\ \vdots \\ V_N \end{bmatrix} \qquad (2.40)$$

Equation (2.40) is an admittance equivalent of the original network as viewed from the nodes i, i+1, ..., N.

This completes the description of the dynamic equivalencing procedure.

2.12 RESULTS OF STUDY

The system chosen for study as mentioned earlier is the UPSEB system which consists of 13 machines, 71 buses and

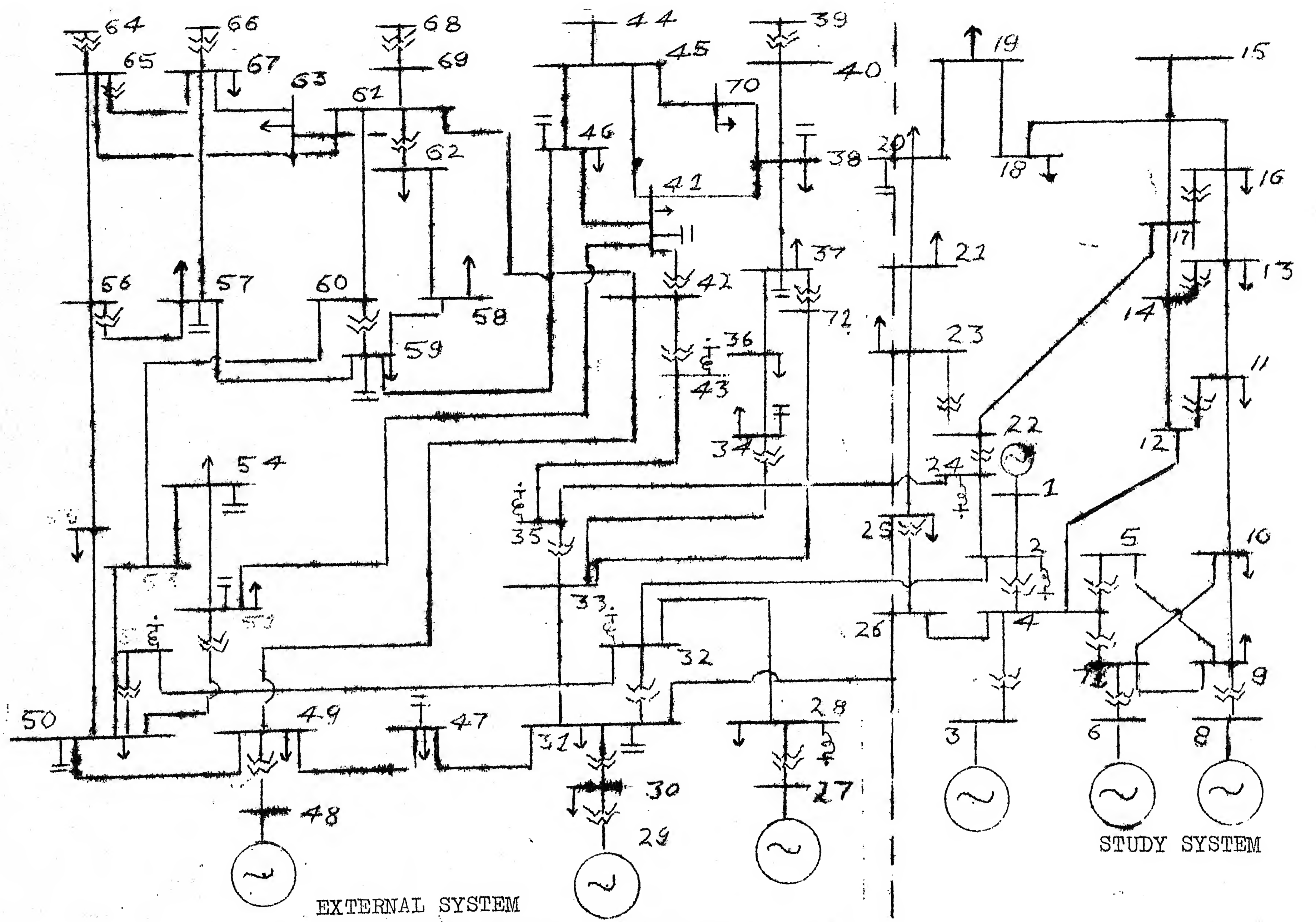

FIG.2.3 : 13-MACHINE SYSTEM SINGLE LINE DIAGRAM

and 94 lines. The single line diagram of the system is given in Fig. 2.3. The data of the system is given in Appendix A.

The results of the dynamic equivalencing will be first presented. The results from coherency analysis show that generators 9, 11, 12, 13 at buses 44, 64, 66 and 68 respectively are coherent.

The bus No.44 is now assumed to be the equivalent bus and the values of the phase-shifting transformer turns ratio and angle are presented in Table 2.1.

TABLE 2.1

FROM BUS	TO BUS	PHASE SHIFTING TRANSFORMER DATA RATIO	ANGLE
45	44	1.05	+0.06108
67	44	1.05	-0.022
65	44	1.05	-0.012
69	44	1.00	-0.0296

The details of the equivalent generator at bus No.44 are presented in Table 2.2.

TABLE 2.2

DETAILS OF EQUIVALENT GENERATOR	VALUE
Total generation	665.0+j180.2451
Total inertia	11.9330
Total reactance	0.0765

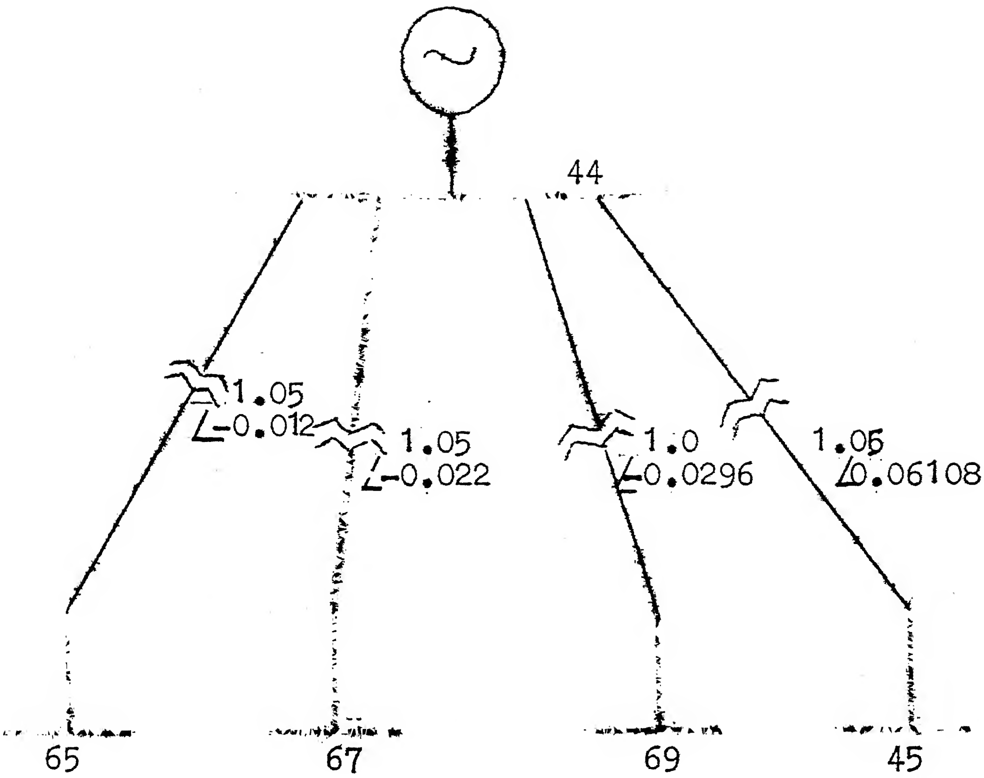

FIG.2.4: GENERATORS 9,11,12,13 AFTER EQUIVALENCING

The change in the physical configuration of the system is as shown in Fig.2.4.

Six representative fault locations are chosen in the study system indicated in Fig.2.3. The critical clearing time for a 3-phase short circuit at these locations is calculated by applying i) Lyapunov's method to the full system, ii) Lyapunov's method to the reduced system after dynamic equivalencing. The results are compared with the clearing time obtained by the step-by-step base case transient stability method. The details are presented in Table 2.3.

TABLE 2.3

METHOD	CLEARING TIME FOR FAULT LOCATIONS IN SECS.					
	Bus 11 Line 8	Bus 10 Line 5	Bus 9 Line 2	Bus 17 Line 27	Bus 25 Line 24	Bus 21 Line 30
1)STEP-BY-STEP BASE CASE TRANSIENT STABILITY	0.23	0.2	0.13	0.16	0.23	0.31
2)LYAPUNOV METHOD FOR THE FULL SYSTEM	0.13	0.10	0.075	0.095	0.13	0.20
3)LYAPUNOV METHOD FOR THE DYNAMIC EQUIVALENT	0.13	0.10	0.075	0.095	0.120	0.180

The computation time taken by the DEC-10 system for the three methods used in calculating the critical clearing time is given

in Table 2.4. The Lyapunov method applied to the dynamic equivalent results in a considerable saving of computer time.

TABLE 2.4

METHOD	COMPUTATION TIME IN SECS
i) STEP-BY-STEP BASE CASE TRANSIENT STABILITY	22
ii) LYAPUNOV METHOD FOR THE FULL SYSTEM	6.64
iii) LYAPUNOV METHOD FOR THE DYNAMIC EQUIVALENT	5.45

2.13 CONCLUSIONS

The Lyapunov function used for the purpose of analysis, was constructed by neglecting the transfer conductances. The results obtained were slightly conservative compared to the actual clearing times

The dynamic equivalent describes the system quite accurately, and is compatible with the Lyapunov's method. The results obtained showed that for fault locations well within the study system the clearing times were very accurate but for fault locations at the boundary of the study system , the clearing times were not the same as those obtained by applying Lyapunov's method to the full system.

There is a large saving in terms of computation time when the Lyapunov's method is used and this may be one of the factors for a trade-off with conservativeness, especially when the Lyapunov's method is used as a preliminary screening procedure to reduce the number of studies to be carried out in detail.

Stature is the charm of a man
Character is the charm in stature
Knowledge is the charm in character
And forgiveness the glory of knowledge.

'VEDAS'

CHAPTER III

A NEW DECOMPOSITION STRUCTURE FOR STABILITY ANALYSIS OF LARGE SCALE POWER SYSTEMS USING VECTOR LYAPUNOV FUNCTIONS.

3.1 INTRODUCTION

In this chapter a new decomposition structure for stability analysis of large scale power systems using vector Lyapunov functions is proposed. Both conceptual and numerical difficulties in the analysis of transient stability of multi-machine power systems increase rapidly when the size of the system becomes large [13, 35]. A new approach to this problem, is based on the concept of vector Lyapunov functions and the decomposition - aggregation technique. This new procedure of stability analysis of the complex system is a two level concept. At the outset the composite system is decomposed into simpler subsystems and their interconnections. At the lower level, each of the subsystems is tested for its stability. Scalar Lyapunov functions are constructed and standard techniques are used to estimate stability regions of the low order subsystems. On the higher hierarchial level, an aggregate model involving a vector Lyapunov function is constructed in order to use

estimates of subsystem stability regions and determine an estimate for the stability region of the overall system. This method is suitable for handling large complex power systems due to the following properties:

(i) The method can be carried out systematically to get stability region estimates which can be easily interpreted in terms of system parameters.

(ii) In the course of transient stability analysis, decomposition can be used to take advantage of (or obtain information about) the special structural features of the power systems.

(iii) The method opens up a real possibility for more refined models of the subsystems to be included in the analysis of large power systems. This is based upon the fact that when more complete descriptions of the individual machines are used; it is easier to construct appropriate Lyapunov functions for pairs of machines than to find a single Lyapunov function for the entire system.

Application of the decomposition - aggregation method to the transient stability problem of large scale power systems was initiated by Pai and Narayana [51]. Their results were obtained in the right direction towards evolving an effective decomposition procedure, but their technique had one drawback. They decomposed a n-machine power system into n(n-1)/2 subsystems

thus making the number of subsystems exceed the order of the system whenever the number of machines is larger than three. Jocic & Siljak [53] and Jocic, Ribbens - Pavella & Siljak [52], decomposed a n-machine power system into n-1 subsystems, but their method suffers from one limitation i.e. a reference machine n should be so chosen that $\frac{D_n}{M_n} > \frac{D_i}{M_i}$ $i = 1,2,\ldots,n-1$, where D is the damping coefficient and M the inertia constant. Thus the decomposition structure hinges on the choice of the reference machine.

In this chapter a new decomposition structure is proposed, based on the centre of angle principle originated by Stanton [54] and Tavora and Smith [55]. An appropriate subsystem Lyapunov function is constructed using the method outlined in [52]. Using the separate results of ref. [41,45,48,56, 57] stability region estimates are computed by a quadratic Lyapunov function. The technique is illustrated for a 4 machine numerical example detailed in Ref. [8].

3.2 DECOMPOSITION OF A MULTIMACHINE POWER SYSTEM

The swing equations for the postfault system characterizing a n-machine power system as indicated in equations (2.5) of Chapter II are given by

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i = - \sum_{\substack{j=1 \\ j \neq i}}^{n} E_i E_j Y_{ij} [\operatorname{Cos}(\delta_{ij} - \theta_{ij}) - \operatorname{Cos}(\delta_{ij}^{o} - \theta_{ij})]$$

$$i = 1,2,\ldots,n \qquad (3.1)$$

Following the method proposed by Stanton [54] and Tavora and Smith [55] the motion of the generator rotors can be resolved into two components:

(i) Motion of the 'Centre of inertia' defined by

$$\delta_o = \frac{1}{M_T} \sum_{i=1}^{n} M_i \, \delta_i \tag{3.2}$$

where $M_T = \sum_{i=1}^{n} M_i$

(ii) The relative motion between the centre of inertia and individual generators, known as the off-centre angles and denoted by

$$\delta_{io} = \delta_i - \delta_o \qquad i = 1,2,\ldots,n \tag{3.3}$$

The off-centre angles are not all independent and have the following properties

$$\sum_{i=1}^{n} M_{i} \, \delta_{io} = 0 \qquad \text{(a)}$$

$$\sum_{i=1}^{n} M_i \, \dot{\delta}_{io} = 0 \qquad \text{(b)} \tag{3.4}$$

$$\sum_{i=1}^{n} M_i \, \ddot{\delta}_{io} = 0 \qquad \text{(c)}$$

The equations of motion in the new co-ordinates are obtained as follows [58].

From equation (2.1) of Chapter II

$$M_i \, \ddot{\delta}_i = - D_i \, \dot{\delta}_i + P_{mi} - P_{ci} \qquad i=1,2,\ldots,n \tag{3.5}$$

From eqn. (3.2)

$$M_T \ddot{\delta}_o = \sum_{i=1}^{n} M_i \ddot{\delta}_i = -\sum_{i=1}^{n} D_i \dot{\delta}_i + \sum_{i=1}^{n} (P_{mi} - P_{ei}) \tag{3.6}$$

From the R.H.S. of eqn. (3.6) adding and subtracting $\sum_{i=1}^{n} D_i \dot{\delta}_o$ we get

$$\ddot{\delta}_o = \frac{D_T}{M_T} \dot{\delta}_o - \frac{\sum_{i=1}^{n} D_i \dot{\delta}_{io}}{M_T} + \frac{\sum_{i=1}^{n} P_{mi}}{M_T} - \frac{\sum_{i=1}^{n} P_{ei}}{M_T} \tag{3.7}$$

where $D_T = \sum_{i=1}^{n} D_i$

The linear dependence that exists among the off-centre variables indicated in (3.4(a)), reveals that it is always possible to describe relative motions within the system through n-1 off centre angles. Manipulation of equations (3.5) and (3.7) yields a set of differential equations which explicitly characterizes the internal motions of the system. These equations are obtained as follows:

Adding and subtracting $D_i \dot{\delta}_o$ from the R.H.S. of equation (3.5) yields

$$M_i \ddot{\delta}_i = -D_i(\dot{\delta}_i - \dot{\delta}_o) - D_i \dot{\delta}_o + P_{mi} - P_{ei} \quad i = 1,2,\ldots,n \tag{3.8}$$

Subtracting equation (3.7) from (3.8) yields

$$\ddot{\delta}_{io} = -\left(\frac{D_i}{M_i} \dot{\delta}_{io} - \frac{\sum_{i=1}^{n} D_i \dot{\delta}_{io}}{M_T}\right) - \left(\frac{D_i}{M_i} - \frac{D_T}{M_T}\right) \dot{\delta}_o + \left(\frac{P_{mi}}{M_i} - \frac{\sum_{i=1}^{n} P_{mi}}{M_T}\right) - \left(\frac{P_{ei}}{M_i} - \frac{\sum_{i=1}^{m} P_{ei}}{M_T}\right) \quad i=1,2,\ldots,n-1 \tag{3.9}$$

The set of equations (3.7) and (3.9) are equivalent to the original system description provided by (3.1). In the new set, however, the equations are expressed in terms of variables which have excellent properties for a proper casting of the state space form.

The subsystems are now cast in the state space form. To aid in the suitable selection of a subsystem nonlinearity the n^{th} machine is taken as the reference machine without loss of generality. Using the fact that $\dot{\delta}_{in} = \dot{\delta}_{io} - \dot{\delta}_{no}$ and equation (3.4(b)), results in equation (3.10(a)). Equation (3.10(b)) follows directly from equation (3.9). Because of the appearance of $\dot{\delta}_{o}$ in the state space equations, we attach equation (3.7) to each of the subsystem descriptions. The resulting state space model for each subsystem is given by

$$\dot{\delta}_{in} = \dot{\delta}_{io} - \dot{\delta}_{no} = \omega_{io} - \omega_{no} = [1+M_i M_n^{-1}]\omega_{io} + M_n^{-1} \sum_{\substack{j=1 \\ j\neq i}}^{n-1} M_j \omega_{jo} \qquad \text{(a)}$$

$$\dot{\omega}_{io} = -\left(\frac{D_i}{M_i} - \frac{(D_i - D_n M_i M_n^{-1})}{M_T}\right)\omega_{io} + M_T^{-1} \sum_{\substack{j=1 \\ j\neq i}}^{n-1} (D_j - D_n M_j M_n^{-1})\omega_{jo}$$

$$- \left(\frac{D_i}{M_i} - \frac{D_T}{M_T}\right)\omega_o - M_i^{-1} \sum_{\substack{j=1 \\ j\neq i}}^{n} A_{ij} f_{ij} + 2M_T^{-1} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ij} \text{Cos}\theta_{ij}$$

$$[\text{Cos}\, \delta_{ij} - \cos \delta^{e}_{ij}] \qquad \text{(b)}$$

$$\dot{\omega}_o = -\left(\frac{D_i - D_n M_i M_n^{-1}}{M_T}\right)\omega_{io} - M_T^{-1} \sum_{\substack{j=1 \\ j\neq i}}^{n-1} (D_j - D_n M_j M_n^{-1})\,\omega_{jo} - \frac{D_T}{M_T}\,\omega_o$$

$$- 2\, M_T^{-1} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ij} \operatorname{Cos} \theta_{ij} \left[\operatorname{Cos} \delta_{ij} - \operatorname{Cos} \delta_{ij}^{o}\right] \quad \text{(c)} \quad (3.10)$$

where

$\dot{\delta}_{io} = \omega_{io}$, $\dot{\delta}_o = \omega_o$, $A_{ij} = E_i E_j Y_{ij}$ and $f_{ij} = \operatorname{Cos}(\delta_{ij} - \theta_{ij}) - \operatorname{Cos}(\delta_{ij}^{o} - \theta_{ij})$

In (3.10) δ_{ij}^{o}'s correspond to the postfault stable equilibrium state given by equation (3.1). Defining an incremental state variable $\delta_{in} - \delta_{in}^{o}$, the state-vector for each subsystem becomes

$$x_i = (\delta_{in} - \delta_{in}^{o}, \omega_{io}, \omega_o)^T \qquad (3.11)$$

The components of the subsystem state vector are the relative rotor angles (any machine being chosen as the reference machine), relative velocity between machine i and centre of angle velocity and centre of angle velocity itself. This decomposition structure differs from that in Refs. [52 53] and in particular the subsystem has a simpler structure. The interconnection terms though different reduce to those of Jocic and Siljak [53] at the aggregation stage.

With the choice of the state vector as in (3.11) and expanding $f_{ij} = \operatorname{Cos}(\delta_{ij} - \theta_{ij}) - \operatorname{Cos}(\delta_{ij}^{o} - \theta_{ij})$ trigonometrically, equation (3.10) can be cast in the form:

$$\dot{\underline{x}}_i = \begin{bmatrix} 0 & (1+M_iM_n^{-1}) & 0 \\ 0 & -\left(\frac{D_i}{M_i} - \frac{(D_i - D_nM_iM_n^{-1})}{M_T}\right) & -\left(\frac{D_i}{M_i} - \frac{D_T}{M_T}\right) \\ 0 & -\left(\frac{D_i - D_nM_iM_n^{-1}}{M_T}\right) & -\frac{D_T}{M_T} \end{bmatrix} \underline{x}_i + \begin{bmatrix} 0 \\ -b_i \\ 0 \end{bmatrix} \phi_i(\underline{y}_i) + h_i(\underline{x})$$

$$\underline{y}_i = [1 \quad 0 \quad 0]\, \underline{x}_i \quad ; \qquad i = 1,2,\ldots,n-1 \qquad (3.12)$$

where

$$\underline{x} = (\delta_{1n} - \overset{o}{\delta}_{1n}, \ldots, \delta_{n-1,n} - \overset{o}{\delta}_{n-1,n}, \omega_{10}, \ldots \omega_{n-1,o}, \omega_o)^T \qquad (3.13)$$

is the state vector of dimension (2n-1) for the overall system (3.12)

$$\phi_i(\underline{y}_i) = \text{Sin}(y_i + \overset{o}{\delta}_{in}) - \sin \overset{o}{\delta}_{in} \qquad (3.14)$$

forms the subsystem nonlinearity.

$$h_i(\underline{x}) = \begin{bmatrix} M_n^{-1} \sum\limits_{\substack{j=1 \\ j\neq i}}^{n-1} M_j \omega_{jo} \\ -b_i l_i \cot \theta_{in} - M_i^{-1} \sum\limits_{\substack{j=1 \\ j\neq i}}^{n-1} A_{ij} f_{ij} + M_T^{-1} \sum\limits_{\substack{j=1 \\ j\neq i}}^{n-1} (D_j - D_nM_jM_n^{-1})\omega_{jo} \\ +2M_T^{-1} \sum\limits_{i=1}^{n-1} \sum\limits_{j=i+1}^{n} A_{ij} \text{ Cos } \theta_{ij} [\text{Cos } \delta_{ij} - \text{Cos } \overset{o}{\delta}_{ij}] \\ -2M_T^{-1} \sum\limits_{i=1}^{n-1} \sum\limits_{j=i+1}^{n} A_{ij} \text{ Cos } \theta_{ij} [\text{Cos } \delta_{ij} - \text{Cos } \overset{o}{\delta}_{ij}] \\ -M_T^{-1} \sum\limits_{\substack{j=1 \\ j\neq i}}^{n-1} (D_j - D_nM_jM_n^{-1})\omega_{jo} \end{bmatrix} \qquad (3.15)$$

and $l_i = \cos\delta_{in} - \cos\delta^{o}_{in}$, $b_i = M_i^{-1} A_{in} \sin\theta_{in}$

For a clearer understanding the decomposition is illustrated for a 3-machine system, with machine 3 as reference. There will be two subsystems given by:

SUBSYSTEM 1

$$\dot{\underline{x}}_1 = \begin{bmatrix} 0 & (1+M_1M_3^{-1}) & 0 \\ 0 & -\left(\frac{D_1}{M_1} - \frac{(D_1-D_3M_1M_3^{-1})}{M_T}\right) & -\left(\frac{D_1}{M_1} - \frac{D_T}{M_T}\right) \\ 0 & -\left(\frac{D_1-D_3M_1M_3^{-1}}{M_T}\right) & -\frac{D_T}{M_T} \end{bmatrix} \underline{x}_1 + \begin{bmatrix} 0 \\ -b_1 \\ 0 \end{bmatrix} \phi_1(\underline{y}_1) + h_1(\underline{x})$$

$$\underline{y}_1 = [1 \quad 0 \quad 0]\,\underline{x}_1 \tag{3.16}$$

SUBSYSTEM 2

$$\dot{\underline{x}}_2 = \begin{bmatrix} 0 & (1+M_2M_3^{-1}) & 0 \\ 0 & -\left(\frac{D_2}{M_2} - \frac{(D_2-D_3M_2M_3^{-1})}{M_T}\right) & -\left(\frac{D_2}{M_2} - \frac{D_T}{M_T}\right) \\ 0 & -\left(\frac{D_2-D_3M_2M_3^{-1}}{M_T}\right) & -\frac{D_T}{M_T} \end{bmatrix} \underline{x}_2 + \begin{bmatrix} 0 \\ -b_2 \\ 0 \end{bmatrix} \phi_2(\underline{y}_2) + h_2(\underline{x})$$

$$\underline{y}_2 = [1 \quad 0 \quad 0]\,\underline{x}_2 \tag{3.17}$$

This completes the decomposition procedure.

3.3 SUB SYSTEM ANALYSIS

Each free subsystem is given by eqn.(3.18) obtained from (3.12) by neglecting the interconnection terms.

$$\dot{\underline{x}}_i = \begin{bmatrix} 0 & (1+M_iM_n^{-1}) & 0 \\ 0 & -(\frac{D_i}{M_i} - \frac{(D_i - D_nM_iM_n^{-1})}{M_T}) & -(\frac{D_i}{M_i} - \frac{D_T}{M_T}) \\ 0 & -(\frac{D_i - D_nM_iM_n^{-1}}{M_T}) & -\frac{D_T}{M_T} \end{bmatrix} \underline{x}_i + \begin{bmatrix} 0 \\ -b_i \\ 0 \end{bmatrix} \phi_i(\underline{y}_i) \tag{3.18}$$

$$\underline{y}_i = [1 \quad 0 \quad 0]\, \underline{x}_i$$

where $\underline{x}_i = (x_{i1}, x_{i2}, x_{i3})^T$ is the state vector defined in (3.9) with $x_{i1} = \delta_{in} - \delta^o_{in}$, $x_{i2} = \omega_{io}$ and $x_{i3} = \omega_o$. $\phi_i(y_i)$ is specified in (3.14). The objective of this section is to construct a Lure'-Postnikov type of Lyapunov function using Kalman's construction. The free subsystem (3.18) should be first transformed to obtain a subsystem with an asymptotically stable linear part. In other words, we add to the right side of the second equation in (3.18) the zero term $\alpha b_i \underline{y}_i - \alpha b_i [0 \quad 1] x_i$ where $b_i = M_i^{-1} A_{in}$ $\mathrm{SIN}\theta_{in}$ and α is a positive number. After a simple regrouping of the terms, we get the transformed subsystem as

$$\begin{aligned} \dot{\underline{x}}_i &= A_i \underline{x}_i + B_i \phi_i(\underline{y}_i) \\ \underline{y}_i &= C_i^T \underline{x}_i \qquad i = 1,2,\ldots,n-1 \end{aligned} \tag{3.19}$$

where

$$A_i = \begin{bmatrix} 0 & (1+M_iM_n^{-1}) & 0 \\ -\alpha b_i & -\left(\frac{D_i}{M_i} - \frac{(D_i - D_nM_iM_n^{-1})}{M_T}\right) & -\left(\frac{D_i}{M_i} - \frac{D_T}{M_T}\right) \\ 0 & -\left(\frac{D_i - D_nM_iM_n^{-1}}{M_T}\right) & -\frac{D_T}{M_T} \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ -b_i \\ 0 \end{bmatrix}, \quad C_i^T = \begin{bmatrix} \theta & 1 \end{bmatrix} \tag{3.20}$$

Kalman's construction procedure for the above system is detailed as follows.

1) The transfer function of the linear part of the system under consideration can be written in terms of the parameters of eqns.(3.19) as

$$W(s) = C_i^T (sI - A_i)^{-1} B_i \tag{3.21}$$

2) Let $Z(s) = (1+\rho_i s) \; W(s)$ (3.22)

so that

$$\tfrac{1}{2}(Z(j\omega) + Z(-j\omega)) = \mathrm{Re}\ (1+\rho_i j\omega)W(j\omega) \tag{3.23}$$

Factorize the expression (3.23) as

$$\tfrac{1}{2}\ (Z(j\omega)+Z(-j\omega)) = m\ (-j\omega)m\ (j\omega) \tag{3.24}$$

3) Determine the scalar

$$\gamma = \rho_i\ (0 + C_i^T B_i) \tag{3.25}$$

4) Solve the identity

$$m(s) - \sqrt{\gamma} = -g_i^T (sI - A_i)^{-1} B_i \tag{3.26}$$

for the components of the vector $\underline{g_i}$.

5) Solve the Lyapunov matrix equation

$$A_i^T H_i + H_i A_i = - g_i g_i^T \qquad (3.27)$$

for the symmetric positive definite matrix $H_i > 0$. A point to be noted is that γ_i is scalar chosen so as to satisfy the Popov's frequency criterion

$$\text{Re } (1+ \gamma_i j\omega) \, W(j\omega) > 0$$

The Lyapunov function is given by

$$V_i(x_i) = x_i^T \underline{H}_i x_i + \gamma_i \int_0^{C_i^T x_i} \phi_i(y_i)\, d y_i \qquad (3.28)$$

where $\underline{H}_i$ is obtained from H_i in (3.27) by making $\alpha = 0$.
The function $V_i(x_i)$ has the estimates

$$\begin{aligned} \eta_{i1} \|x_i\|^2 \leq V_i(x_i) &\leq \eta_{i2} \|x_i\|^2 \\ \dot{V}_i(x_i)_{(3.18)} &\leq - \eta_{i3} \|x_i\| \end{aligned} \qquad (3.29)$$

as shown by Weissenberger [56].

Jocic and Siljak [53] have shown that

$$\eta_{i1} = \lambda(\underline{H}_i) \qquad \eta_{i2} = \Lambda(\overline{H}_i) \text{ and } \eta_{i3} = \lambda(G_i) \quad i=1,2,\ldots,n-1 \qquad (3.30)$$

where λ and Λ are the minimum and maximum eigenvalues of the corresponding matrices. $\underline{H}_i$ is obtained from H_i by setting $\alpha=0$ in (3.27) and $\overline{H}_i = \underline{H}_i + \frac{1}{2} \gamma_i K_i' C_i C_i^T$. The choice of $\overline{H}_i$ comes from the fact that $y_i \phi_i(y_i) \leq K_i' y_i^2$ where K_i' is the slope of the function $\phi_i(y_i)$ at $y_i=0$, and the fact that the integral in

$V_i(x_i)$ of (3.28) can be majorized by $\frac{1}{2} K_i' y_i^2$.

To derive the second inequality in (3.29) $\dot{V}_i(x_i)_{(3.18)}$ is first computed. A positive number ε_i is so chosen that the inequality

$$x_{i1}\phi_i(x_{i1}) \geqslant \varepsilon_i x_{i1}^2 \qquad (3.31)$$

is satisfied in an interval $[x_{i1}', x_{i1}'']$, where the limits x_{i1}' and x_{i1}'' are the nonzero solutions of the equation

$$\varepsilon_i x_{i1} = \text{Sin}\,(x_{i1} + \delta_{in}^o) - \text{Sin}\,\delta_{in}^o \qquad (3.32)$$

using (3.31) G_i is calculated from the inequality

$$\dot{V}_i(x_i)_{(3.18)} \leqslant -x_i^T G_i x_i \qquad (3.32)$$

The estimates (3.29) are needed in constructing an aggregate model for the system (3.12). The stability region estimate $\tilde{X}_i$ for each decoupled system (3.18) can be readily computed using the results of Walker and McClamroch [59] and Weissenberger [56] as

$$\tilde{X}_i = \left\{ x_i : V_i(x_i) < V_i^o \right\} \; ; \quad i=1,2,\ldots,n-1 \qquad (3.33)$$

where

$$V_i^o = \min_{y_i = x_{i1}', x_{i1}''} \left\{ \left[\left(\frac{D_i}{M_i} - \frac{(D_i - D_n M_i M_n^{-1})}{M_T} \right) - \gamma_i^{-1} \right] y_i^2 + \gamma_i b_i \left[\text{Cos}\,\delta_{in}^o - \text{Cos}(y_i + \delta_{in}^o) - y_i \text{Sin}\,\delta_{in}^o \right] \right\} \qquad (3.34)$$

and x_{i1}', x''_{i1} are the solutions of (3.32). This completes the subsystem analysis.

3.4 STABILITY REGION ESTIMATES FOR OVERALL SYSTEM

An estimate $\tilde{X}$ of the stability region X for the overall system (3.12) is defined.

DEFINITION

A region $\tilde{X}$ of points $X \in R^m$ where R^m is the m-dimensional Euclidean space , is an estimate of the region of asymptotic stability $X(\tilde{X} \subset X)$ for the equilibrium x=0 if and only if x=0 is stable in the sense of Lyapunov, $(t_o, x_o) \in T \times \tilde{X}$ where T represents the time interval, implies $x(t; t_o, x_o) \subset \tilde{X}$ for all $t \in T_o$ where T_o is the semi-infinite time interval $(t_o, +\infty)$ and $\lim_{t \to \infty} x(t; t_o, x_o) = 0$.

To compute X defined above a quadratic Lyapunov function is used

$$\nu(x) = V^T D V \tag{3.35}$$

where the vector Lyapunov function is defined as

$$V(x) = [V_1^{\frac{1}{2}}(x_1), V_2^{\frac{1}{2}}(x_2), \ldots, V_{n-1}^{\frac{1}{2}}(x_{n-1})]^T \tag{3.36}$$

and matrix D has the diagonal form

$$D = \text{diag}\{d_1, d_2, \ldots, d_{n-1}\}, \tag{3.37}$$

with $d_i > 0$, $i=1,2,\ldots, n-1$

The desired asymptotic property of $\tilde{X}$ is established by the stability of a $(n-1) \times (n-1)$ matrix $W = (w_{ij})$ which is defined by

$$w_{ij} = \begin{cases} -\lambda^{-1}(H_i)\,\lambda(G_i) + & i = j \\ \lambda^{-\frac{1}{2}}(H_i)\,\lambda^{-\frac{1}{2}}(H_j)\,\xi_{ij} & i \neq j \end{cases} \tag{3.38}$$

where ξ_{ij} is the interconnection constant. The procedure to determine matrices $\underline{H}_i$, $\overline{H}_i$ and G_i is given in section 3.3.

The region

$$\tilde{X} = \left\{ x : \nu(x) < \nu^o \right\} \tag{3.39}$$

with $$\nu^o = \min_{1 \leq i \leq n-1} \left\{ d_i V_i^o \right\} \tag{3.40}$$

and $$\nu(x) = \sum_{i=1}^{n-1} d_i V_i(x_i) \tag{3.41}$$

is an estimate of the stability region X containing the equilibrium x=0, of the system described in (3.12), if the matrix W defined by equation (3.38) satisfies the inequalities

$$(-1)^k \begin{vmatrix} w_{11} & w_{12} & \cdots & w_{1k} \\ w_{21} & w_{22} & \cdots & w_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ w_{k1} & w_{k2} & \cdots & w_{kk} \end{vmatrix} > 0, \forall\, k=1,2,\ldots,n-1 \tag{3.42}$$

To determine the interconnection constants ξ_{ij}'s* the following procedure is adopted. $\dot{\nu}(x)_{(3.12)}$ is computed as

$$\dot{\nu}(x)_{(3.12)} = \sum_{i=1}^{n-1} d_i \left\{ \dot{V}_i(x_i)_{(3.18)} + [\text{grad } V_i(x_i)]^T h_i(x) \right\} \qquad \textbf{(a)}$$

$$\leq \sum_{i=1}^{n-1} d_i \left[-(G_i) \|x_i\|^2 + \sum_{\substack{j=1 \\ j\neq i}}^{n-1} 2\,\xi_{ij} \|x_i\| \|x_j\| \right] \qquad \textbf{(b)}$$

(3.43)

comparing the two equations in the set (3.43) ξ_{ij}'s are obtained.

***Details given in Appendix B.**

Now using equations (3.34), (3.39) and (3.40) the stability region estimate can be computed. This completes the procedure for computing an estimate for the stability region of the overall multimachine power system.

3.5 NUMERICAL EXAMPLE

In this section, an illustration of the proposed approach is presented using a realistic four machine power system. The data and details of the system are available in ref. [8].

$M_1 = 75.35, \quad M_2 = 1.130, \quad M_3 = 2.260, \quad M_4 = 1.508$

$D_1 = 1, \quad D_2 = 12 \quad D_3 = 2.5 \quad D_4 = 6.0$

$$\left.\begin{array}{lll} Y_{21} = 0.66416 & Y_{31} = 0.66098 & Y_{41} = 0.7606 \\ \theta_{21} = 91.294^\circ & \theta_{31} = 97.039^\circ & \theta_{41} = 99.152^\circ \\ \delta^\circ_{21} = 5.85^\circ & \delta^\circ_{31} = 13.79^\circ & \delta^\circ_{41} = 2.96^\circ \end{array}\right\} \text{Details Refer to postfault stable equilibrium point} \tag{3.44}$$

Machine 1 is chosen as the reference machine. The positive numbers $\int_i$ and ε_i are chosen as follows:

$$\int_2 = \int_3 = \int_4 = 1; \quad \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0.5 \tag{3.45}$$

Equation (3.32) is solved for $y_i = x_{i1}$ to get the domain for subsystem nonlinearities as

$$\begin{array}{ll} x_2' = 99.05^\circ, & x_2'' = -117.9^\circ \\ x_3' = 85.7^\circ, & x_3'' = -130.074^\circ \\ x_4' = 103.8^\circ, & x_4'' = -113.34^\circ \end{array} \tag{3.46}$$

which are used to compute V_i^o in (3.34) for the three decoupled subsystems.

The minimum and maximum eigenvalues, which are needed for aggregation, are found to be

$$\lambda(\underline{H}_2) = 0.361 \quad , \Lambda(\overline{H}_2) = 157.765 \quad , \quad \lambda(G_2) = 1.389$$
$$\lambda(\underline{H}_3) = 0.00651 \quad , \Lambda(\overline{H}_3) = 567.651 \quad , \quad \lambda(G_3) = 0.16517$$
$$\lambda(\underline{H}_4) = 0.02399 \quad , \Lambda(\overline{H}_4) = 231.136 \quad , \quad \lambda(G_4) = 0.25554 \tag{3.47}$$

By using equation (3.43), the bounds on the nonlinearities are calculated as

$$\begin{aligned}
\xi_{22} &= 0.1627 & \xi_{23} &= 0.000065 & \xi_{24} &= 0.000024 \\
\xi_{32} &= 0.000065 & \xi_{33} &= 0.0957 & \xi_{34} &= 0.00001312 \\
\xi_{42} &= 0.000024 & \xi_{43} &= 0.00001312 & \xi_{44} &= 0.0819
\end{aligned} \tag{3.48}$$

The aggregate matrix

$$W = \begin{bmatrix} -0.00882 & 0.00134029 & 0.00025776 \\ 0.00134029 & -0.0002906 & 0.0001005 \\ 0.00025776 & 0.0001005 & -0.001105 \end{bmatrix} \tag{3.49}$$

satisfies the inequalities (3.42) and the matrix D of (3.37) with the choice

$$d_2 = d_3 = d_4 = 1$$

From equation (3.34)

$$V_2^o = 29.29, \; V_3^o = 0.43758, \quad V_4^o = 10.16816, \quad \mu^o = 0.43758 \tag{3.50}$$

Using equations (3.39), (3.40) and (3.41) we get the stability region estimate as

$$
\begin{aligned}
V(x) = {} & 5.11(\delta_{21}-\delta_{21}^{o})^2 + \tfrac{1}{2}\omega_{20}(\delta_{21}-\delta_{21}^{o}) - 0.27\omega_{o}(\delta_{21}-\delta_{21}^{o}) + \tfrac{1}{2}(\delta_{21}-\delta_{21}^{o})\omega_{20} \\
& +0.49\omega_{20}^{2} - 3.64\omega_{o}\omega_{20} - 0.27(\delta_{21}-\delta_{21}^{o})\omega_{o} - 3.64\omega_{20}\omega_{0} \\
& +157.38\omega_{o}^{2} + 0.92\,(0.99-(\delta_{21}-\delta_{21}^{o})0.101 - \cos\delta_{21}) + \\
& .52(\delta_{31}-\delta_{31}^{o})^2 + \tfrac{1}{2}\omega_{30}(\delta_{31}-\delta_{31}^{o}) - 0.057\omega_{o}(\delta_{31}-\delta_{31}^{o}) + \tfrac{1}{2}(\delta_{31}-\delta_{31}^{o})\omega_{30} \\
& +1.004\omega_{30}^{2} - 17.10\omega_{o}\omega_{30} - 0.057(\delta_{31}-\delta_{31}^{o})\omega_{o} - 17.10\,\omega_{30}\omega_{0} \\
& +567.13\omega_{0}^{2} + 0.34\,(0.97 - (\delta_{31}-\delta_{31}^{o})0.23 - \cos\delta_{31}) + \\
& 1.93(\delta_{41}-\delta_{41}^{o})^2 + \tfrac{1}{2}\omega_{40}(\delta_{41}-\delta_{41}^{o}) - 0.13\omega_{0}(\delta_{41}-\delta_{41}^{o}) + \tfrac{1}{2}(\delta_{41}-\delta_{41}^{o})\omega_{40} \\
& +0.55\omega_{40}^{2} - 9.67\omega_{o}\omega_{40} - 0.13(\delta_{41}-\delta_{41}^{o})\omega_{0} - 9.67\omega_{40}\omega_{0} \\
& +230.73\omega_{0}^{2} + 0.53\,(0.99 - (\delta_{41}-\delta_{41}^{o})\,0.051 - \cos\delta_{41}) \\
& < 0.43758
\end{aligned}
\tag{3.51}
$$

The faulted system equations are then integrated from the prefault equilibrium state, till the region of stability is reached. This gives the critical clearing time. The critical clearing time obtained for this example is

$$t_c = 0.22 \text{ sec} \tag{3.52}$$

as compared to $t_c = 0.4244$ seconds, obtained by applying Lyapunov's direct method as in ref. [8]. This completes the numerical example for the 4 machine system. Although the results are somewhat conservative, this represents the first application

of the vector Lyapunov function approach to a realistic power system to compute critical clearing time. Selection of parameters in (3.45) was somewhat arbitrary. An optimum choice of these parameters will lead to a reduction in the conservativeness. This is an open area for research.

3.6 CONCLUSIONS

This chapter has demonstrated the applicability of the vector Lyapunov functions to the stability analysis of large scale power systems. The key to a successful application of the method lies in an effective decomposition of the power system. In this chapter a new decomposition procedure is developed. This enables one to construct Lure' Postikov type Lyapunov functions. Using these functions a scalar Lyapunov function for the overall system is obtained via a vector Lyapunov function. The critical clearing time is then evaluated. Although the clearing time obtained is quite conservative, the possibility of reducing conservativeness by including more system details makes this new approach an attractive tool for a detailed stability analysis of large scale power systems.

There are no eyes that see like knowledge;
there are no joys as in truthfulness;
there are no pains as in attachment;
and there is no bliss as in desirelessness.

'VEDAS'

CHAPTER IV

INCLUSION OF NON-NEGLIGIBLE TRANSFER CONDUCTANCES IN LYAPUNOV FUNCTIONS FOR MULTIMACHINE POWER SYSTEMS

4.1 INTRODUCTION

In the transient stability analysis of a power system via the direct method of Lyapunov a suitable mathematical model of the system is a basic requirement. For multimachine systems this means that, in addition to the governor and exciter dynamics, the damper windings in the direct and quadrature axis, in the dynamic model of the synchronous machine, be taken into account. This necessitates the use of Park's equations, change of reference axis between machine and network reference frames etc., resulting in a mathematical model involving complicated nonlinearity. Analysis by Lyapunov's method therefore becomes practically impossible. Hence, a number of simplifying assumptions have hitherto been made so that the mathematical model is amenable to analysis by Lyapunov based methods. These assumptions have been detailed in section (2.2) of Chapter II.

Efforts to relax these assumptions have met with limited success. In particular, there have been efforts [6,60,14,12] to relax the assumption regarding the neglect of transfer conductances. The transfer conductances are off diagonal conductance terms in the admittance matrix reduced at the generator internal nodes. Their magnitude depends on the network parameters of the system, location of the fault and loading conditions. Although the conductance components take small values as compared with susceptance components, it still remains an open problem whether or not we can neglect the conductance components from the standpoint of the transient stability. For some cases, the neglect of transfer conductances may be justified, as shown by Ribbens-Pavella [12]. However, if the system is heavily loaded, it has been pointed out by Uyemura et al [60] that there exists a danger of judging a practically unstable system to be stable if we use a Lyapunov function which neglects the transfer conductances.

In this chapter a method proposed by Pai and Varwandkar [62] has been used to construct Lyapunov functions which include effects of transfer conductances. Computations of critical clearing time for a practical 13 machine 71 bus power system are done, and compared with those obtained in Chapter II by neglecting the transfer conductances.

4.2 MATHEMATICAL MODEL AND LYAPUNOV FUNCTIONS

The model of a n-machine power system with transfer conductances can be written as

$$\dot{x} = Ax - B_1 f(\sigma) - B_2 g(\sigma) \tag{4.1}$$
$$\sigma = Cx$$

where A, B_1, B_2 and C are constant matrices, x is a $(2n-1)$-dimensional state vector and the $n(n-1)/2$- dimensional non-linearity vectors $f(\sigma)$ and $g(\sigma)$ are such that their i^{th} component depends on only the i^{th} component of σ ; i.e. $f_i(\sigma) = f_i(\sigma_i)$ and $g_i(\sigma) = g_i(\sigma_i)$.

Without transfer conductances, however, the model becomes

$$\dot{x} = Ax - B_1 f(\sigma)$$
$$\sigma = Cx \tag{4.2}$$

Typically, for a three-machine system with machine 1 as reference, we have

$$x = [\omega_1 \;\; \omega_2 \;\; \omega_3 \;\; (\delta_{12} - \overset{o}{\delta}_{12}) \;\; (\delta_{13} - \overset{o}{\delta}_{13})]^T$$

$$A = \left[\begin{array}{ccc|cc} -\frac{D_1}{M_1} & 0 & 0 & 0 & 0 \\ 0 & -\frac{D_2}{M_2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{D_3}{M_3} & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \end{array}\right] \qquad B_1 = \left[\begin{array}{ccc} \frac{1}{M_1} & \frac{1}{M_1} & 0 \\ -\frac{1}{M_2} & 0 & \frac{1}{M_2} \\ 0 & -\frac{1}{M_3} & -\frac{1}{M_3} \\ \cdots & \cdots & \cdots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$B_2 = \begin{bmatrix} \frac{1}{M_1} & \frac{1}{M_1} & 0 \\ \frac{1}{M_2} & 0 & \frac{1}{M_2} \\ 0 & \frac{1}{M_3} & \frac{1}{M_3} \\ \cdots & \cdots & \cdots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad C = \left[\begin{array}{ccc|cc} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$f_1(\sigma_1) = E_1E_2B_{12}\ [\operatorname{Sin}(\sigma_1 + \delta_{12}^{o}) - \operatorname{Sin}\delta_{12}^{o}]$$

$$f_2(\sigma_2) = E_1E_3B_{13}\ [\operatorname{Sin}(\sigma_2 + \delta_{13}^{o}) - \operatorname{Sin}\delta_{13}^{o}]$$

$$f_3(\sigma_3) = E_2E_3B_{23}\ [\operatorname{Sin}(\sigma_3 + \delta_{23}^{o}) - \operatorname{Sin}\delta_{23}^{o}]$$

and similarly

$$g_1(\sigma_1) = E_1E_2G_{12}\ [\operatorname{Cos}(\sigma_1 + \delta_{12}^{o}) - \operatorname{Cos}\delta_{12}^{o}]$$

$$g_2(\sigma_2) = E_1E_3G_{13}\ [\operatorname{Cos}(\sigma_2 + \delta_{13}^{o}) - \operatorname{Cos}\delta_{13}^{o}]$$

$$g_3(\sigma_3) = E_2E_3G_{23}\ [\operatorname{Cos}(\sigma_3 + \delta_{23}^{o}) - \operatorname{Cos}\delta_{23}^{o}] \tag{4.3}$$

where the double scripted B's and G's are the transfer susceptance and transfer conductance terms, respectively.

The nonlinearity $f_i(\sigma_i)$ satisfies the sector condition

$$0 < \frac{f_i(\sigma_i)}{\sigma_i} < K, \qquad K = \infty \tag{4.4}$$

in the interval

$$-\pi - 2\delta_{pq}^{o} < \sigma_i < \pi - 2\delta_{pq}^{o} \tag{4.5}$$

where p and q ($q > p$) are the indices of the angles at the generator internal nodes on which the i^{th} nonlinearity depends.

The nonlinearity $g_i(\sigma_i)$, unfortunately, does not satisfy a similar sector condition. One can easily establish that near the origin it is in the first and third quadrants if G_{pq} Sin $\delta^o_{pq} < 0$ and in the second and fourth quadrants if G_{pq} Sin $\delta^o_{pq} > 0$ for all $q > p$. Because of this uncertainty in the sign of this nonlinearity, the system (4.1) cannot be cast in the Lure' form, and therefore the systematic procedure given by Pai and Murthy [12] and Willems [26] cannot be applied to form a Lyapunov function.

Now consider the nonlinearity $h_i(\sigma_i)=f_i(\sigma_i)+g_i(\sigma_i)$

$$= E_p E_q Y_{pq} [\mathrm{Sin}(\sigma_i + \delta^o_{pq} + \theta_{pq}) - \mathrm{Sin}(\delta^o_{pq} + \theta_{pq})]$$

$$G_{pq} = Y_{pq} \,\mathrm{Sin}\, \theta_{pq}$$

$$B_{pq} = Y_{pq} \,\mathrm{Cos}\, \theta_{pq} \tag{4.6}$$

This nonlinearity satisfies the Popov sector condition in the interval.

$$-\pi - 2(\delta^o_{pq} + \theta_{pq}) < \sigma_i < -2(\delta^o_{pq} + \theta_{pq}) \tag{4.7}$$

Introducing this nonlinearity, system (4.1) can be rewritten as

$$\dot{x} = Ax - B_1 h(\sigma) - [B_2 - B_1] g(\sigma)$$
$$\sigma = Cx \tag{4.8}$$

This particular representation has certain advantages over (4.1) and (4.2) in that the nonlinearity $h_i(\sigma_i)$ contains

transfer conductance terms and still satisfies the sector condition. Also, it is possible to show that it is the last term i.e. $[B_2-B_1]\ g(\sigma)$ in (4.8) which presents difficulty in forming the Lyapunov function, and if neglected, the Lyapunov function can easily be constructed.

The Lyapunov function for (4.2), i.e. the system neglecting transfer conductances [12], [15] is given by

$$V(x,\sigma) = x^T Px + 2\int_0^{Cx} f^T(\sigma)Q\,d\sigma \tag{4.9}$$

where Q = identity matrix, with a negative semidefinite derivative along (4.2) given by

$$\dot{V}(x,\sigma) = -x^T LL^T x \tag{4.10}$$

where matrices L & P ($P > 0$) satisfy the equations

$$A^T P + PA = -LL^T$$

and

$$PB_1 = A^T C^T Q \tag{4.11}$$

The validity of the Lyapunov function (4.9) for the model (4.1) can be considered by differentiating (4.9) along (4.1). Recognizing the fact that $CB_2=0$

$$\dot{V}(x,\sigma) = \dot{V}(x,\sigma) - 2x^T PB_2 g(\sigma) \tag{4.12}$$

It is possible to show via a three-machine example that certain terms in (4.12) cannot be integrated to form a suitable Lyapunov function. For a three-machine system [15]

$$P = \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 \\ 0 & 0 & M_2 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.13}$$

so that

$$\tilde{\dot{V}}(x, \sigma) = \dot{V}(x, \sigma) - 2\left[(\omega_1+\omega_2)g_1(\sigma_1)+(\omega_1+\omega_3)g_2(\sigma_2)+(\omega_2+\omega_3)g_3(\sigma_3)\right]$$

Since $\dot{\sigma}_1 = \omega_1 - \omega_2$, $\dot{\sigma}_2 = \omega_1 - \omega_3$ and $\dot{\sigma}_3 = \omega_2 - \omega_3$,

$$\tilde{\dot{V}}(x, \sigma) = \dot{V}(x, \sigma) - 2g_1(\sigma_1)\dot{\sigma}_1 - 2g_2(\sigma_2)\dot{\sigma}_2 - 2g_3(\sigma_3)\dot{\sigma}_3$$
$$-4\left[\omega_2 g_1(\sigma_1) + \left\{g_2(\sigma_2) - g_3(\sigma_3)\right\}\omega_3\right] \tag{4.14}$$

In (4.14) all terms except $\dot{V}(x, \sigma)$ are sign indefinite. Hence, almost every research work in this area so far has preferred to ignore the terms containing $g(\sigma)$ in (4.1). However, the second, third and fourth terms in (4.14), although sign indefinite, can be integrated and subtracted from (4.9) to give

$$V_c(x, \sigma) = x^T Px + 2\int_0^{Cx} f^T(\sigma)Qd\sigma + 2\int_0^{Cx} g^T(\sigma)Qd\sigma$$
$$= x^T Px + 2\int_0^{Cx} h^T(\sigma)Qd\sigma \tag{4.15}$$

which is positive definite around the origin in view of (4.6) and (4.7).

The last term in (4.14), which cannot be integrated directly is, in fact, the expanded form of

$$2x^T P\,[B_2 - B_1]\,g(\sigma) \tag{4.16}$$

Since (4.8) is equivalent to (4.1) and noting the structure of (4.16), it can be asserted that (4.15) constitutes a valid Lyapunov function for the approximate model obtained from (4.8) by deleting the term $[B_2-B_1]\,g(\sigma)$, i.e. for the system

$$\begin{aligned} \dot{x} &= Ax - B_1 h(\sigma) \\ \sigma &= Cx \end{aligned} \tag{4.17}$$

which is in the Lure' form.

In (4.17) all the transfer conductances appear in $h(\sigma)$, and hence the critical clearing time computed on the basis of the Lyapunov function (4.15) is expected to be closer to that obtained by the actual simulation of the swing curves. This completes the mathematical modelling and construction of the Lyapunov function.

4.3 RESULTS OF STUDY

The study is carried out on the system detailed in Appendix A. The same set of representative fault locations, as those used in Chapter II are taken to obtain an effective comparison. Three phase faults are considered at the above mentioned locations and critical clearing time is calculated using Lyapunov's method for the model including transfer conductances and the results are compared with those obtained in Chapter II by neglecting the transfer conductances. Table 4.1 effectively summarises the results and it is seen that the

estimate obtained by including transfer conductances is slightly on the higher side and is much closer to the actual value obtained by the step-by-step base case transient stability method.

TABLE 4.1

METHOD	CLEARING TIME FOR FAULT LOCATIONS IN SECS.					
	Bus 11 Line 8	Bus 10 Line 5	Bus 9 Line 2	Bus 17 Line 27	Bus 25 Line 24	Bus 21 Line 30
1) STEP-BY-STEP BASE CASE TRANSIENT STABILITY	0.23	0.2	0.13	0.16	0.23	0.31
2) LYAPUNOV METHOD NEGLECTING TRANSFER CONDUCTANCES	0.130	0.100	0.075	0.095	0.130	0.20
3) LYAPUNOV METHOD INCLUDING TRANSFER CONDUCTANCES	0.145	0.120	0.085	0.105	0.145	0.21

4.4 CONCLUSION

In this chapter the problem of transfer conductances in Lyapunov functions for multimachine power systems has been discussed. A method of accounting the transfer conductances, proposed by Pai and Varwandkar [62] has been used, on a realistic system. The results obtained, show that including the transfer

conductances, gives higher clearing times, closer to the actual values, thus establishing the superiority of the Lyapunov function based on the model (4.17).

Man makes the cosmos and its construction the pivot of his emotional life in order to find in this way the peace and serenity which he cannot find in the narrow whirlpool of personal experience... The supreme task ... is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intution resting on sympathetic understanding of experience can reach them ...

'EINSTEIN'

CHAPTER V

CONCLUSIONS

5.1 SUMMARY AND CONCLUSIONS

The development of Lyapunov theory for the stability analysis of power systems has been carried out with sufficient interest in recent years. The method has proven to be an effective alternative to the simulation techniques both practically and theoretically. Considerable interest has been shown by research workers as far as the construction of Lyapunov functions is concerned, but the application of the method to realistic power systems has been slow. Apart from the inherent conservativeness of the method, the size of the system and detailed modelling have been the major stumbling blocks in the successful application of this method. Hence, alternate approaches are called for to bypass and eliminate these draw-backs. Vector Lyapunov functions and dynamic equivalencing are

two of the most promising solutions for the above mentioned problems.

In this thesis, stress has been laid on some aspects of the above two problems. The following are some of the conclusions arrived at.

In Chapter II, a coherency based dynamic equivalent is obtained for a practical electric utility in India. Lyapunov's method is applied to the equivalent and critical clearing time is calculated for various fault locations. The results are compared with those obtained by applying Lyapunov's method to the full system. Although the results may be conservative, the dynamic equivalent gives clearing times which tally with the actual system everywhere except at the boundary of the study system. It is also to be noted that compared to using Lyapunov's method for the full system the method using dynamic equivalents results in a 20% saving in computation time. This saving will be much more for bigger systems. Thus the method is attractive for planning purposes and transient security analysis using Lyapunov's method.

Chapter III presents a new decomposition structure, based on the centre of angle principle, for vector Lyapunov functions. The vector Lyapunov function approach is a two level concept and utilizes the decomposition-aggregation technique

for the stability analysis of large scale power systems. Decomposition of the system is a key step in the application of these concepts. The decomposition is pairwise in nature, hence it is possible to include system refinements in each of these subsystems at the lower level. Studies were carried out on a realistic four machine system and even though the critical clearing time obtained was conservative, the possibility of including system refinements clearly outweighs this disadvantage. Ways of reducing the conservative nature have been pointed.

The presence of transfer conductances in multimachine power systems and how to accommodate it in the Lyapunov function is the topic of Chapter IV. This fits into our philosophy of trying to make the Lyapunov method an effective practical tool, by taking into account a more detailed representation of the power system. An approximate model including the effects of transfer conductances is used. Calculation of critical fault clearing times in a realistic 13 machine 71 bus system demonstrates the advantage of including transfer conductances.

5.2 SUGGESTIONS FOR FUTURE RESEARCH

On the basis of the investigation carried out in this thesis, some of the unsolved problems that need further attention are:

1. In Chapter II further investigation is necessary regarding

inclusion of voltage regulator dynamics, as well as other refinements like saturation, in the Lyapunov analysis.

2. The decomposition achieved in Chapter III uses only the classical model. An important feature of the new method is its ability to handle complexity of the power systems. This opens up a real possibility for introducing more refined mathematical models of multimachine systems. Also one can examine newer decomposition methods such as those using Diakoptic techniques.

3. Computer algorithms can be used to determine the parameters $\mathcal{f}_i$, ε_i and provide an optimal choice of the parameters $\mathcal{f}_i$ and ε_i with respect to the size of the estimate $\tilde{X}$ for the overall stability region.

4. Inclusion of transfer conductances, in an exact manner, in the mathematical model for obtaining suitable Lyapunov functions for power systems is still open for further research. The model used in Chapter IV, should be exploited further for modifying the Lure' type Lyapunov function with the deleted terms taken as perturbations in the model.

LIST OF REFERENCES

1 S.B. CRARY: Power System Stability, Vol.I & Vol.II Wiley, New York, 1945, 1947.

2 E.W. KIMBARK: Power System Stability, Vol.I (Elements of Stability Calculations), Wiley, New York, 1948.

3 E.W. KIMBARK: Power System Stability, Vol.III (Synchronous Machines), Wiley, New York, 1956.

4 H.P. HERTLEIN: 'Critique on the state of the art in power system stability studies', Ph.D. Thesis Lehigh University, 1967.

5 N. DHARMA RAO: 'A new approach to the transient stability problem', AIEE TRANS. Power Apparatus and Systems Vol.81, Oct.1962, pp 186-192.

6 P.D Aylett: 'The energy integral criterion of transient stability limits of power systems', Proc. IEE, Vol.105(c), July 1958, pp 527-536.

7 G.E. GLESS: 'Direct method of Lyapunov applied to transient power system stability', IEEE Trans. P.A.S., Vol.PAS-85, Feb. 1966, pp 164-179.

8 A.H. EL-ABIAD & K. NAGAPPAN: 'Transient stability regions of multimachine power systems', IEEE Trans. PAS, Vol.PAS-85 Feb.1966 pp 158-168.

9 M.A. PAI: 'Transient stability studies in power systems using Lyapunov-Popov approach' - Paper No.265, presented at the 5th IFAC CONGRESS, Paris, June 12-17, 1972.

10 M.A. PAI & M.A. MOHAN: 'Transient stability of multimachine power systems by Popov's method' Paper no.70CP667-PWR, presented at the IEEE Summer Power Meeting and EHV Conference, Los Angeles, July 1970.

11 M.A. PAI, M. ANANDA MOHAN & J. GOPALA RAO: 'Power system transient stability regions using Popov's method', IEEE TRANS. PAS, Vol.PAS-89, May/June 1970, pp.788-794.

12 M.A. PAI & P.G. MURTHY: 'New Lyapunov functions for power systems based on minimal-realizations', Int. J. of Control, Vol.19, No.2 Feb. 1974, pp 401-415.

13 M. RIBBENS PAVELLA: Survey of studies on transient power system stability by Lyapunov's direct method', Proceeding of Seminar on 'Stability of Large Scale Power Systems at the University of Liege, Liege, Belgium, June 19-22, 1972. Publisher: Western Periodicals Co., North Hollywood, California 91605.

14. M. RIBBENS PAVELLA: 'Transient stability of multimachine power systems by Lyapunov's direct method', Paper No.71CP17 PWR Presented at the IEEE Power Engg. Society Winter Power Meeting, New York Jan. 1971.

15 J.C. WILLEMS & J.L. WILLEMS: 'The application of Lyapunov methods to the computations of transient stability regions for multimachine power systems' IEEE TRANS. PAS Vol.PAS-89 May/June 1970, pp 795-801.

16 G.A. LUDERS: 'Transient stability of multimachine power systems via the direct method of Lyapunov', IEEE TRANS. on PAS, Vol.PAS-90 Jan/Feb. 1971, pp 23-36.

17 M. MANSOUR: 'Generalized Lyapunov functions for power systems', IEEE TRANS. on Aut. Control (Corresp) Vol.AC-19, No.3 June 1974, pp 247-248.

18 M. ANANDA MOHAN: 'Power system transient stability by methods of Popov and Lyapunov', Ph.D. thesis submitted to the Indian Institute of Technology, Kanpur (INDIA), June 1970.

19 P.G. MURTHY: 'Transient stability investigation of power systems using improved Lyapunov functions', Ph.D. thesis submitted to the Indian Institute of Technology, Kanpur (INDIA), Sept. 1973.

20 UYEMARA & MATSUKI: 'A computational algorithm for evaluating unstable equilibrium-points in power-systems', Electrical Engg. in Japan, July/Aug 1972, pp 41-47.

21 F.S. PRABHAKARA & A.H. EL_ABIAD: 'A simplified determination of transient stability regions for Lyapunov methods', IEEE TRANS. Pas, Vol.PAS-94, March/April 1975, pp 672-689.

22 M.A. PAI & C.L. NARAYANA: 'Finite regions of attraction for multinonlinear system and an application to the power system stability problem', IEEE TRANS on Aut. Control, 21, 1976 pp.716-720.

23 C.L. NARAYANA: 'Stability of Large Scale Power Systems' Ph.D. thesis submitted to the Indian Institute of Technology, Kanpur (INDIA) Dec. 1975.

24 A BHARGAVA: 'Application of Lyapunov's Method to Large Power Systems' M.Tech thesis submitted to the Indian Institute of Technology, Kanpur (INDIA) Aug. 1975.

25 S.D. VARWANDKAR: 'Stability Investigations in Multimachine Power Systems' Ph.D thesis submitted to the Indian Institute of Technology, Kanpur (INDIA) July 1977.

26 J.L. WILLEMS: 'Optimum Lyapunov functions and stability regions for multimachine power systems', Proc IEE Vol.117, No.3, March 1970, pp 573-577.

27 J.P. FANTIN: 'L' Application de la methode directe de Lyapunov a l'etude de la stabilite dyamique des reseause d' energie electrique'- Ph.D. thesis submitted to AL' universite paul saliatier de tenlouse (France) April 1974.

28 F. SACCOMANO: 'Global methods of stability assessment' paper presented at the Symposium on 'Power System Dynamics at the University of Manchester, Sept 1973.

29 R. W. deMELLO, R. PODMORE & K.N. STANTON 'Coherency based dynamic equivalents: Applications in Transient Stability Studies', PICA Conference Proc., 1975 pp 23-31.

30 SYSTEMS CONTROL INC: 'Coherency based dynamic equivalents for transient-stability studies', Final report on EPRI Project RP-90-4, Phase II, Jan. 1975.

31 SYSTEMS CONTROL INC: 'Development of Dynamic Equivalents for Transient stability studies', Final report on EPRI EL456, Project 763, May 1977.

32 M.A. PAI: CHAPTER In Hand book of large scale system engineering application Edited by M.G. singh and A. Title. North Holland Publishing Co, Amsterdam 1979.

33 M.A. PAI & R.P. ADGAONKAR 'Identification of Coherent generators using weighted eigenvectors' paper No.A-990225 presented at IEEE PES Winter meeting, New York Feb.1979.

34 R. Podmore 'Identification of coherent generators for dynamic equivalents', Paper No.F77155-5, IEEE PES Winter power meeting, New York, Jan. 1977.

35 A.A. FOUAD, 'Stability theory-Criteria for transient stability' in Proc. Conf. on Syst. Eng. for Power: Status and Prospects Henniker, N.H., 1975 pp 421-450 .

36 F.N. BAILEY: 'The application of Lyapunov's second method to interconnected systems', Journal SIAM (Control) Ser.A, Vol3, No.3, 1966, pp 443-462.

37 R. BELLMAN: 'Vector Lyapunov functions', Journal SIAM (Contro Ser.A. Vol.1, 1962, pp 32-34.

38. F.C. SCHWEPPE & S.J. MITTER: 'Hierarchial system theory, Proceedings of the symposium of real time control of electrical systems (Book), Editor E. Handschin, Elsevier Publishing Co. 1972 pp 259-265.

39 PIONTKOVSKII & L.D. RUTKOVSKAYA: 'Investigation of certain stability theory problems by vector Lyapunov functions', Automation & Remote Control, Oct. 1967 pp 1422-1429.

40 D.D. SILJAK: 'Stability of large scale systems under structural perturbations', IEEE Trans. on Systems-Man and Cybernetics, Vol.SMC-2, No.5, Nov.1972 pp 657-663.

41 D.D. SILJAK: 'Stability of large scale systems', Proc. of the 5th IFAC Congress, Paris, June 1972.

42 D.D. SILJAK' On large scale system stability' Paper presented at the 9th Allerton Conference on Circuits and System theory, University of Illinois, Oct6-8, 1971.

43 D.D. SILJAK: 'Large-Scale Systems Stability & Structure' North-Holland, New York.

44 Lj.T. GRUJIC & D.D. SILJAK: 'Stability of large scale systems with stable and unstable systems, Proc. of Joint Aut. Control conference California 1972, pp 550-555.

45 Lj.T. GRUJIC & D.D. SILJAK: 'Asymptotic, Stability and instability of large scale systems' IEEE Trans. on Aut. control, Vol. AC-18, No.6, Dec.1973, pp 636-645.

46 W.E. THOMPSON: 'Exponential stability of interconnected systems', IEEE Trans. on Aut. Control (Corresp) Vol.AC-15, Aug.1970, pp 504-506.

47 M. ARAKI & B. KONDO: 'Stability & transient behaviour of composite nonlinear systems', IEEE Trans. on Aut. control, Vol.AC-17 Aug. 1972, pp 537-541.

48 A.N. MICHEL: 'Stability analysis of interconnected systems', J. Siam Control, Vol.12, No.3, Aug. 1974, pp 554-579.

49 A.N. Michel: 'Stability analysis and trajectory behaviour of composite systems', IEEE Trans. on circuits and systems, Vol.22, No.4, April 1975, pp 305-312.

50 A.N. MICHEL & R.K. MILLER: 'Qualitative analysis of large scale dynamical systems', (Book) Academic Press, 1977.

51 M.A. PAI & C.L. NARAYANA: 'Stability of large scale power systems' in Proc. 6th IFAC Congr. Boston, MA, 1975 pp.31.6:1-10.

52 Lj.B. JOCIC, M. RIBBENS-PAVELLA & D.D. SILJAK: 'Multimachine Power Systems: Stability, Decomposition and Aggregation', IEEE Trans. On Aut. Control, Vol.AC-23, No.2 April 1978. pp 325-332.

53 Lj.B. JOCIC & D.D. SILJAK: 'Decomposition and Stability of Multimachine power systems' presented at 7th IFAC World Congress Helsinki, Finland, June 1978.

54 K.N. STANTON: 'Dynamic energy balance studies for simulation of power frequency transients', IEEE trans. Vol.PAS-91, pp 110-118, Jan.1972.

55 C.J. TAVORA & O.J.M. SMITH: 'Characterization of equilibrium and Stability in power systems', IEEE Trans. Vol.PAS-91, pp. 1127-1130, No.3, 1972.

56 S. Weissenberger: 'Stability regions of large scale systems.' Automatica 9, 1973 pp.653-663.

57 M. ARAKI: 'Application of M-matrices to the stability problems of composite dynamical systems J. Math. and Appl. 52, 1975, pp 309-321.

58 K.R. PADIYAR & C.V. DORESWAMY, 'Analysis of Dynamic equilibrium of Power system', 7th Annual sourth eastern symposium on system theory March 20-21, 1975, pp 123-125.

59 J.A. WALKER & N.H. McCLAMROCH: 'Finite regions of Attractions for the problem of Lure'' Int. J. Control, 6, 1967, pp.331-336.

60 K. UEMARA, J. MATASUKI, I. YAMADA & T. TSUJI: 'Approximation of an energy function in transient stability analysis of power systems', Elec. Engg. Japan, Vol.92 No.6, Nov/Dec. 1972 pp 96-100.

61 S. KITAMURA, T. DOHNOMOTO & Y. KUREMATSU: 'Construction of a Lyapunov function by the perturbation method and its application to the transient stability problem of power systems with non-negligible transfer conductances' Int. J. control Vol.26, 1977 pp. 405-420.

62 M.A. PAI & S.D. VARWANDKAR: 'On the inclusion of transfer conductances in Lyapunov functions for multimachine power system' IEEE Trans. On Aut. Control (Corres.), Vol.AC-22, No.6, Dec.1977, pp.983-985.

APPENDIX A

DATA FOR 13 M/C, 71 BUS SYSTEM

Number of busses = 71 Number of machines = 13

Number of lines = 94 Number of shunt loads = 23

Base power (MVA) = 200

Bus No.	Name	Generation		Load Power		Bus voltages Mag.	Ang
1	OBRAEXT11	630.6572	169.7222	0.0000	0.0000	1.0300	0.000
2	OBRAEXT400	0.0	0.0	0.0	0.0	1.0577	-5.931
3	OBRA(T)11	506.0000	149.4929	0.0	0.0	1.0250	-1.406
4	OBRA(T)220	0.0	0.0	0.0	0.0	1.0545	-6.061
5	OBRA(T)132	0.0	0.0	0.0	0.0	1.0476	-5.064
6	OBRA(H)11	98.0000	31.9867	0.0	0.0	1.0250	0.397
7	OBRA(H)132	0.0	0.0	12.8	8.3	1.0458	-5.723
8	RIH11	297.0	124.2276	0.0	0.0	1.0250	-0.204
9	RIH132	0.0	0.0	185.0	130.0	1.0436	-4.976
10	ROB132	0.0	0.0	80.0	50.0	1.0268	-7.413
11	MUG132	0.0	0.0	155.0	96.0	1.0237	-11.762
12	MUG220	0.0	0.0	0.0	0.0	1.0030	-10.057
13	MAU132	0.0	0.0	100.0	62.0	1.0072	-13.829
14	MAU220	0.0	0.0	0.0	0.0	1.0041	-11984
15	GOR11	184.08	113.0	0.0	0.0	1.0055	-9.413
16	GOR132	0.0	0.0	73.0	45.5	1.0248	-16.682
17	GOR220	0.0	0.0	36.0	22.4	1.0051	-14.460

Bus No.	Name	Generation		Load power		Bus voltages Mag.	Ang.
18	KHAL132	0.0	0.0	16.0	9.0	1.0097	-17.881
19	BAST132	0.0	0.0	32.0	19.8	0.9812	-19.786
20	GON132	0.0	0.0	27.0	16.8	1.0030	-21.838
21	FAI132	0.0	0.0	32.0	19.8	1.0168	-21.376
22	SUL220	0.0	0.0	0.0	0.0	1.0082	-17.625
23	SUL132	0.0	0.0	75.0	46.6	1.0361	-19.815
24	SUL400	0.0	0.0	0.0	0.0	0.9859	-16.193
25	ALL132	0.0	0.0	133.0	82.5	1.0391	-17.441
26	ALL220	0.0	0.0	0.0	0.0	1.0160	-14.382
27	MAT11	304.0	76.2874	0.0	0.0	1.0250	-5.453
28	MAT400	0.0	0.0	30.0	20.0	1.0522	-12.342
29	PANK11	261.0	70.5069	0.0	0.0	1.0250	-15.519
30	PANK132	0.0	0.0	120.0	74.5	1.0574	-19.947
31	PANK220	0.0	0.0	160.0	99.4	1.0148	-22.817
32	PANK400	0.0	0.0	0.0	0.0	1.0252	-15.936
33	LUCK220	0.0	0.0	0.0	0.0	0.9984	-25.515
34	LUCK132	0.0	0.0	112.0	69.5	1.3066	-29.639
35	LUCK400	0.0	0.0	0.0	0.0	0.9664	-21.803
36	SIT132	0.0	0.0	50.0	32.0	0.9943	-34.030
37	SHA132	0.0	0.0	147.0	92.0	0.9673	-37.982
38	BAR132	0.0	0.0	93.5	88.0	0.9725	-37.638
39	KHAT11	25.0	30.3833	0.0	0.0	1.0250	-34.166
40	KHAT132	0.0	0.0	0.0	0.0	1.0183	-36.913

Bus No.	Name	Generation		Load power		Bus voltages Mag.	Ang.
41	MOR132	0.0	0.0	255.0	123.0	1.0074	-34.778
42	MOR220	0.0	0.0	0.0	0.0	0.9791	-31.481
43	MOR400	0.0	0.0	0.0	0.0	0.9462	-27.835
44	RAM11	180.0	55.0370	0.0	0.0.0	1.0250	-24.448
45	RAM132	0.0	0.0	0.0	0.0	1.0483	-30.514
46	NEH132	0.0	0.0	78.0	38.6	1.0200	-32.678
47	MAI220	0.0	0.0	234.0	145.0	0.9893	-29.053
48	HARJ11	341.0	256.0	0.0	0.0	1.0051	-25.781
49	HARJ220	0.0	0.0	295.0	183.0	0.9972	-30.500
50	MUR220	0.0	0.0	40.0	24.6	0.9776	-32.056
51	MUR132	0.0	0.0	227.0	142.0	1.0051	-35.309
52	MUR400	0.0	0.0	0.0	0.0	0.9579	-26.552
53	MEER220	0.0	0.0	0.0	0.0	0.9723	-32.845
54	MEER132	0.0	0.0	108.0	68.0	1.0049	-35.318
55	SHAM220	0.0	0.0	25.5	48.0	0.9852	-31.313
56	SAHA220	0.0	0.0	0.0	0.0	1.0130	-29.253
57	SAHA132	0.0	0.0	55.6	35.6	1.0162	-29.967
58	HARD132	0.0	0.0	42.0	27.0	1.0184	-30.261
59	ROO132	0.0	0.0	57.0	27.4	1.0135	-30.715
60	ROO220	0.0	0.0	0.0	0.0	1.0087	-29.974
61	RIS220	0.0	0.0	0.0	0.0	1.0197	-27.874
62	RIS132	0.0	0.0	40.0	27.0	1.0443	-28.940
63	DEH132	0.0	0.0	33.2	20.6	1.0426	-27.695

Bus No.	Name	Generation		Load power		Bus voltages	
						Mag.	Ang.
64	YAMII11	300.0	72.8823	0.0	0.0	1.0250	-19.401
65	YAMII220	0.0	0.0	0.0	0.0	1.0572	-24.62
66	YAMI,IV11	96.0	25.6366	0.0	0.0	1.0250	-20.610
67	YAMI,IV132	0.0	0.0	14.0	6.5	1.0556	-25.641
68	MAN11	89.0	26.6892	0.0	0.0	1.0250	-19.829
69	MAN220	0.0	0.0	0.0	0.0	1.0508	-25.363
70	HALD132	0.0	0.0	11.4	7.0	0.9987	-34.790
71	SHA220	0.0	0.0	0.0	0.0	0.9274	-35.329

GENERATOR DATA

Gen.No.	Bus From	Name	Inertia	Trans.React.	Damp Const.
1	1	OBRAEXT11	31.2500	0.0432	0.0
2	3	OBRA(T)11	15.4050	0.0567	0.0
3	6	OBRA(H)11	1.9230	0.5278	0.0
4	8	RIH11	6.6480	0.2010	0.0
5	15	GOR11	6.2500	0.2160	0.0
6	27	MAT11	12.3750	0.1310	0.0
7	29	PANK11	9.2500	0.1525	0.0
8	39	KHAT11	1.0350	1.0821	0.0
9	44	RAM11	2.2590	0.3520	0.0
10	48	HARJ11	12.6710	0.0648	0.0
11	64	YAMII11	6.1880	0.1500	0.0
12	66	YAMI,IV11	2.2380	0.5280	0.0
13	68	MAN11	1.2480	0.6000	0.0

LINE DATA

Line No.	From Bus	To Bus	Line Impedance		$\frac{1}{2}Y$ charge	Turns Ratio
1	9	8	0.0	0.0570	0.0	1.05
2	9	7	0.0320	0.0780	0.0090	1.00
3	9	5	0.0660	0.1600	0.0047	1.00
4	9	10	0.0520	0.1270	0.0140	1.00
5	10	11	0.0660	0.1610	0.0180	1.00
6	7	10	0.0270	0.0700	0.0070	1.00
7	12	11	0.0	0.0530	0.0	0.95
8	11	13	0.0600	0.1480	0.0380	1.00
9	14	13	0.0	0.0800	0.0	1.00
10	13	16	0.0970	0.2380	0.0270	1.00
11	17	15	0.0	0.0920	0.0	1.05
12	7	6	0.0	0.2220	0.0	1.05
13	7	4	0.0	0.0800	0.0	1.00
14	4	3	0.0	0.0330	0.0	1.05
15	4	5	0.0	0.1600	0.0	1.00
16	4	12	0.0160	0.0790	0.0710	1.00
17	12	14	0.0160	0.0790	0.0710	1.00
18	17	16	0.0	0.0800	0.0	0.95
19	2	4	0.0	0.0620	0.0	1.00
20	4	26	0.0190	0.0950	0.1930	1.00
21	2	1	0.0	0.0340	0.0	1.05
22	31	26	0.0340	0.1670	0.1500	1.00

Line No.	From Bus	To Bus	Line Impedance		½Y charge	Turns ratio
23	26	25	0.0	0.0800	0.0	0.95
24	25	23	0.2040	0.5200	0.0130	1.00
25	22	23	0.0	0.0800	0.0	0.95
26	24	22	0.0	0.0840	0.0	0.95
27	22	17	0.0480	0.2500	0.0505	1.00
28	2	24	0.0100	0.1020	0.3353	1.00
29	23	21	0.0366	0.1412	0.0140	1.00
30	21	20	0.0720	0.1860	0.0050	1.00
31	20	19	0.1460	0.3740	0.0100	1.00
32	19	18	0.0590	0.1500	0.0040	1.00
33	18	16	0.0300	0.0755	0.00801	1.00
34	28	27	0.0	0.0810	0.0	1.05
35	30	29	0.0	0.0610	0.0	1.05
36	32	31	0.0	0.0930	0.0	0.95
37	31	30	0.0	0.0800	0.0	0.95
38	28	32	0.0051	0.0510	0.6706	1.00
39	31	33	0.0130	0.0640	0.0580	1.00
40	31	47	0.0110	0.0790	0.1770	1.00
41	2	32	0.0158	0.1570	0.5100	1.00
42	33	34	0.0	0.0800	0.0	0.95
43	35	33	0.0	0.0840	0.0	0.95
44	35	24	0.0062	0.0612	0.2012	1.00
45	34	36	0.0790	0.2010	0.0220	1.00

Line No.	From Bus	To Bus	Line Impedance		$\frac{1}{2}Y_{charge}$	Turns ratio
46	36	37	0.1690	0.4310	0.0110	1.00
47	37	38	0.0840	0.1880	0.0210	1.00
48	40	39	0.0	0.3800	0.0	1.05
49	40	38	0.0890	0.2170	0.0250	1.00
50	38	41	0.1090	0.1960	0.0220	1.00
51	41	51	0.2350	0.6000	0.0160	1.00
52	42	41	0.0	0.0530	0.0	0.95
53	43	42	0.0	0.0840	0.0	0.95
54	47	49	0.0210	0.1030	0.0920	1.00
55	49	48	0.0	0.0460	0.0	1.05
56	49	50	0.0170	0.0840	0.0760	1.00
57	49	42	0.0370	0.1950	0.0390	1.00
58	50	51	0.0	0.0530	0.0	0.95
59	52	50	0.0	0.0840	0.0	0.95
60	50	55	0.0290	0.1520	0.0300	1.00
61	50	53	0.0100	0.0520	0.0390	1.00
62	53	54	0.0	0.0800	0.0	0.95
63	51	54	0.0220	0.0540	0.0060	1.00
64	55	56	0.0160	0.0850	0.0170	1.00
65	56	57	0.0	0.0800	0.0	1.00
66	57	59	0.0280	0.0720	0.0070	1.00
67	59	58	0.0480	0.1240	0.0120	1.00
68	60	59	0.0	0.0800	0.0	1.00

Line No.	From Bus	To Bus	Line Impedance		½Y charge	Turns ratio
69	53	60	0.0360	0.1840	0.0370	1.00
70	45	44	0.0	0.1200	0.0	1.05
71	45	46	0.0370	0.0900	0.0100	1.00
72	46	41	0.0830	0.1540	0.0170	1.00
73	46	59	0.1070	0.1970	0.0210	1.00
74	60	61	0.0160	0.0830	0.0160	1.00
75	61	62	0.0	0.0800	0.0	0.95
76	58	62	0.0420	0.1080	0.0020	1.00
77	62	63	0.0350	0.0890	0.0090	1.00
78	69	68	0.0	0.2220	0.0	1.05
79	69	61	0.0230	0.1160	0.1040	1.00
80	67	66	0.0	0.1880	0.0	1.05
81	65	64	0.0	0.0630	0.0	1.05
82	65	56	0.0280	0.1440	0.0290	1.00
83	65	61	0.0230	0.1140	0.0240	1.00
84	65	67	0.0240	0.0600	0.0950	1.00
85	67	63	0.0390	0.0990	0.0100	1.00
86	61	42	0.0230	0.2293	0.0695	1.00
87	57	67	0.0550	0.2910	0.0070	1.00
88	45	70	0.1840	0.4680	0.0120	1.00
89	70	38	0.1650	0.4220	0.0110	1.00
90	33	71	0.0570	0.2960	0.0590	1.00
91	71	37	0.0	0.0800	0.0	0.95
92	45	41	0.1530	0.3880	0.0100	1.00
93	35	43	0.0131	0.1306	0.4293	1.00
94	32	52	0.0164	0.1632	0.5360	1.00

SHUNT LOAD DATA

S.No.	Bus No.	Shunt Load	Admittance
1	2	0.0	-0.4275
2	13	0.0	0.1500
3	20	0.0	0.0800
4	24	0.0	-0.2700
5	28	0.0	-0.3375
6	31	0.0	0.200
7	32	0.0	-0.8700
8	34	0.0	0.2250
9	35	0.0	-0.3220
10	36	0.0	0.1000
11	37	0.0	0.3500
12	38	0.0	0.2000
13	41	0.0	0.2000
14	43	0.0	-0.2170
15	46	0.0	0.1000
16	47	0.0	0.3000
17	50	0.0	0.1000
18	51	0.0	0.1750
19	52	0.0	-0.2700
20	54	0.0	0.1500
21	57	0.0	0.1000
22	59	0.0	0.0750
23	21	0.0	0.0500

APPENDIX B

In eqn.3.43(a) and 3.43(b) the first term on the R.H.S. are equal. The second term on the R.H.S. of 3.43(a) when expanded contains terms of the form Sin $\overset{o}{\delta}_{in}\, x_{i1}$, $\text{Sin}(\overset{o}{\delta}_{ij}-\theta_{ij})(x_{i1}+x_{i2})$, and $\text{Sin}(\overset{o}{\delta}_{in}+\theta_{in})\, x_{i1}$ etc. In order to compare with the second term on R.H.S. of 3.43(b) we make use of the following inequalities for the f_{ij}'s defined in eqn. (3.10).

$$f_{ij} \le |\text{Sin } \overset{o}{\delta}_{in}|\,|x_{i1}|$$

$$-f_{ij} \le |\text{Sin } (\overset{o}{\delta}_{ij} - \theta_{ij})|\,(|x_{i1}| + |x_{j1}|)$$

$$f_{nj} \le |\text{Sin } (\overset{o}{\delta}_{jn} + \theta_{jn})|\,|x_{j1}|$$

Also we make use of the triangular inequality

$$x_{i1}+x_{i2}+x_{i3} \le |x_{i1}| + |x_{i2}| + |x_{i3}| \le \sqrt{3}\,\|x_i\|$$

Finally comparison of like terms in R.H.S. of 3.43 (a) and (b) yields

$$\xi_{ij} = \frac{\sqrt{3}}{2}\Big[\,|b_i|\,|\text{Cot}\theta_{in}|\,|\text{Sin } \overset{o}{\delta}_{in}| + \sum_{\substack{k=1\\k\neq i}}^{n-1} M_i^{-1}A_{ik}|\text{Sin}(\overset{o}{\delta}_{ik}-\theta_{ik})|\Big]\; i=j$$

$$\xi_{ij} = \frac{\sqrt{3}}{2}\Big[\,M_i^{-1}A_{ij}|\text{Sin}(\overset{o}{\delta}_{ij}-\theta_{ij}) + M_T^{-1}\sum_{\substack{j=1\\j\neq i}}^{n-1} A_{jn}\text{Sin}(\overset{o}{\delta}_{jn}+\theta_{jn})|\Big]\; i\neq j$$